THE UNIVERSITY OF ALBERTA

SAMPLED DATA CONDITIONAL FEEDBACK SYSTEMS

by

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled Sampled Data Conditional Feedback Systems submitted by Sushil K. Sarna in partial fulfillment of the requirements for the degree of Master of Science.

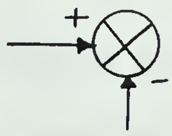
LIST OF SYMBOLS AND ABBREVIATIONS

damping ratio = ζ

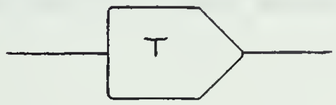
natural frequency of oscillation = ω_n

sampling time period = T

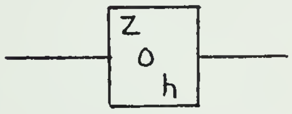
summer =



time delay (T sec.) =



Gho = zero order hold =



EQUIPMENT USED

Analog Computer PACE 231-R

ABSTRACT

This thesis extends the concept of conditional feedback to sampled data systems.

Dead-beat controllers were used to improve the disturbance-output response. The dead-beat controllers were designed by the rep-op method.

Linear and non-linear sampled data systems were studied. The optimisation of oscillatory systems by the use of conditional feedback and the dead-beat controllers was also studied.

ERRATA

Page	Line	Remarks
12	2	read 'checked' instead of 'verified'.
60	11	read 'section' instead of 'article'.

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CHAPTER I

INTRODUCTION

A conditional feedback system is a system in which the feedback path is effective only when there is a disturbance at the plant. In the absence of a disturbance, the conditional feedback system operates as an open loop system.

The concept of conditional feedback was originally put forth by Lang and Ham[1]. They discussed the continuous data conditional feedback systems in their paper.

The basic configuration of a continous data conditional feedback system is shown in fig 1.1. Fig 1.2 shows the corresponding conventional feedback system.

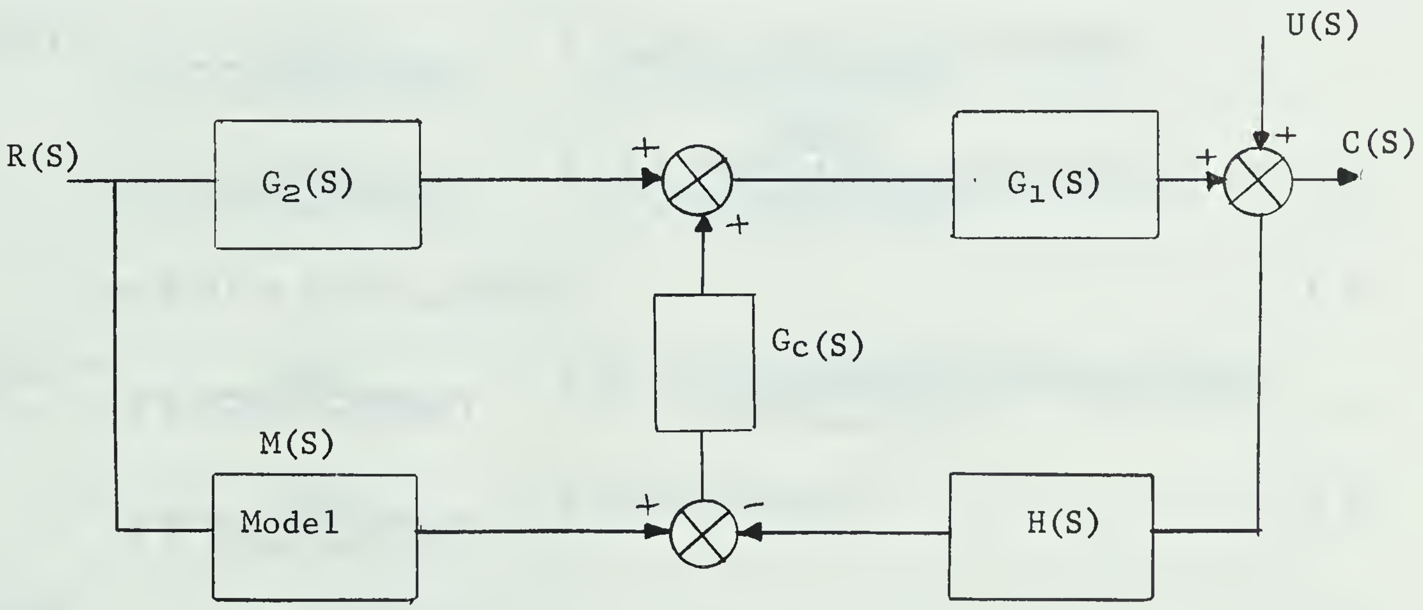


Fig. 1.1 - CONDITIONAL FEEDBACK

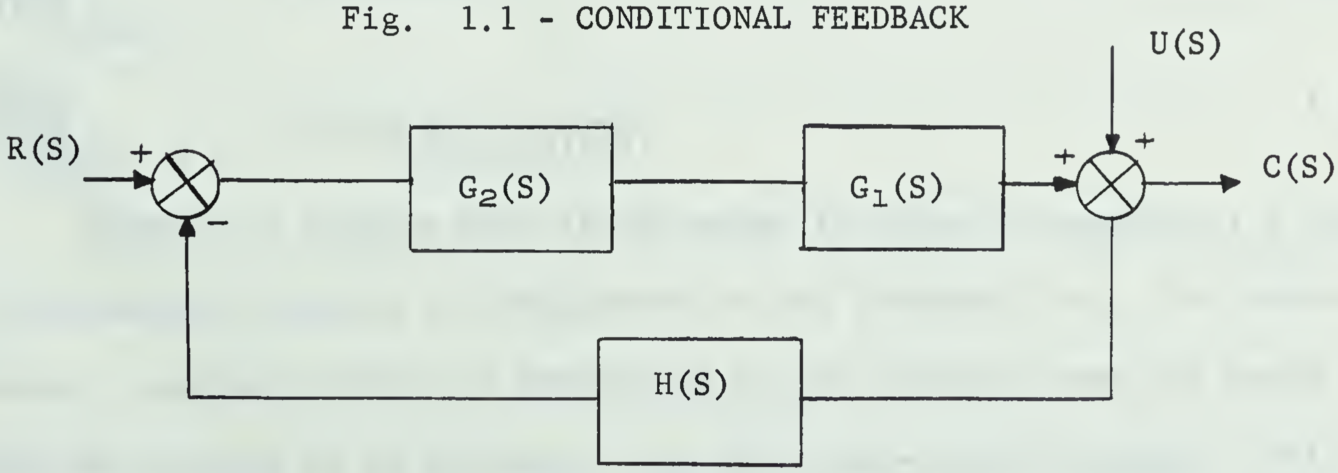


Fig. 1.2 - CONVENTIONAL FEEDBACK

$G_1(S)$ is the Laplace transform of the plant.

$G_2(S)$ is the Laplace transform of a compensator to modify the response of the plant, if desired.

$H(S)$ is the Laplace transform of the sensing device.

$Gc(S)$ is the Laplace transform of the compensator to modify the disturbance-output response. It will be noted later that the input-output response is independent of $G_c(S)$.

$M(S)$ is the Laplace transform of the model to be designed.

$U(S)$ is the disturbance, $R(S)$ the input and $C(S)$ the output.

The Laplace transform of the output can be written as

$$C(S) = \frac{U(S)}{1 + G_1(S)G_c(S)H(S)} + \frac{\{G_2(S) + M(S)Gc(S)\}G_1(S)R(S)}{1 + G_1(S)G_c(S)H(S)}$$

$$= \frac{U(S)}{1 + G_1(S)G_c(S)H(S)} + \frac{\left\{1 + M(S)\frac{Gc(S)}{G_2(S)}\right\}G_1(S)G_2(S)R(S)}{1 + G_1(S)G_c(S)H(S)} \qquad 1.1$$

$$\text{Let } M(S) = G_1(S)G_2(S)H(S) \qquad 1.2$$

$$C(S) = \frac{U(S)}{1 + G_1(S)G_c(S)H(S)} + \frac{\{1 + G_1(S)G_c(S)H(S)\}G_1(S)G_2(S)R(S)}{1 + G_1(S)G_c(S)H(S)}$$

$$= \frac{U(S)}{1 + G_1(S)G_c(S)H(S)} + R(S)G_1(S)G_2(S) \qquad 1.3$$

$$\left.\frac{C(S)}{R(S)}\right|_{U(S) = 0} = G_1(S)G_2(S) \qquad 1.4$$

$$\left.\frac{C(S)}{U(S)}\right|_{R(S) = 0} = \frac{1}{1 + G_1(S)G_c(S)H(S)} \qquad 1.5$$

Equation 1.4 shows that if the model is given by equation 1.2, the input-output response is independent of the feedback loop. The disturbance - output response is determined by the feedback loop and hence it can be designed to be different from the input-output response. This is significant from the design point of view. In the design of a conventional feedback system a compromise has often to be made between the

input-output response and the disturbance - output response. They are basically of the same nature. This problem is largely eliminated by the use of conditional feedback as seen above.

There are, of course, other methods of achieving the same e.g. by the use of auxiliary feedback loops or the use of pre-filters[(1)], but the above method is the most straight forward and requires less additional components.

Another advantage arises from the fact that the system operates as an open loop as far as the input signal is concerned. Thus non-linearities in the plant can be tolerated without causing instability. Similar non-linearities are built in the model for this purpose. If, however, non-linearity in the plant $G_1(S)$ causes instability in the feedback loop when there is a disturbance, then the compensator has to be so designed that the system is stable in the presence of a disturbance also. This modification in the compensator will not affect the input-output response. If it is feasible to place the non-linearity in the branch of the transfer function $G_2(S)$, then the non-linearity will be out of the feedback loop and hence no problem of instability will arise due to it. The input-output response will not be affected by placing the non-linear plant in the branch of the transfer function $G_2(S)$.

Another advantage of operating as an open loop is that low power gains are required.

The open loop system is also advantageous when there are transport delays in the system. These time delays are often a cause of instability in closed loop systems.

The model mentioned above whose transfer function is $G_1(S)G_2(S)H(S)$ will be simulated from simple R-C-L elements and will not be subject to changes except for the tolerance of the components.

The purpose of this thesis is to extend the idea of conditional feedback to sampled data systems and explore the possible improvements by the use of digital controllers.

A secondary object of the thesis is to establish the effectiveness of the Repetive Operation Method (known in short as the Rep-op method, see appendix A), to design dead-beat controllers for linear and non-linear systems. This new, simple, direct and convenient method is due to Professor Y. J. Kingma, who has also designed the equipment that is used for designing the dead-beat controllers by this method.

Most of the design of digital controllers and the study of the response of systems to input signals and disturbances was carried out on the analog computer. The entire work has been divided into four chapters of which the present chapter deals with an introduction to the thesis.

Chapter II deals with linear sampled data systems. It was observed here that, for linear systems, it is possible to simulate a continuous data model. The continuous data model worked well for systems having no inter-sample ripple. In a system with inter-sample ripple, there will be an error at the branch $G_c(S)$ during a sampling period. With the use of conditional feedback, there are possibilities of removing or at least appreciably reducing the inter-sample ripple as discussed in chapter IV. This is achieved by modifying the model.

Continuous data compensators were first tried and they worked well.

The feedback loop in one of these cases was unstable because of high gain. It was stabilized by designing the compensator $G_c(S)$ by the Bode Plot Method (Bi-linear transformation method).

It was observed that a system designed for a step input worked well for a ramp input too. This was also proved analytically.

Dead-beat digital controllers were then tried. The insertion of these controllers did not affect the input-output response but it removed the effect of disturbances on the output in an optimum manner.

Chapter III deals with non-linear sampled data systems. In this case a simple continuous model cannot be found because there is no transform for the non-linearity. The model includes a sampler and a zero order hold.

The dead-beat controllers were designed by the rep-op method. This chapter and the next establish the convenience of the use of rep-op method over other methods. Just like the State Variable Method, the rep-op method has the advantage that there is no inter-sampling ripple, but it is much quicker than the former. The transfer function of the plant used for a saturation type of non-linearity was $\frac{1}{S(S+1)}$ and the non-linearity was located in the positions shown in fig. 1.3.

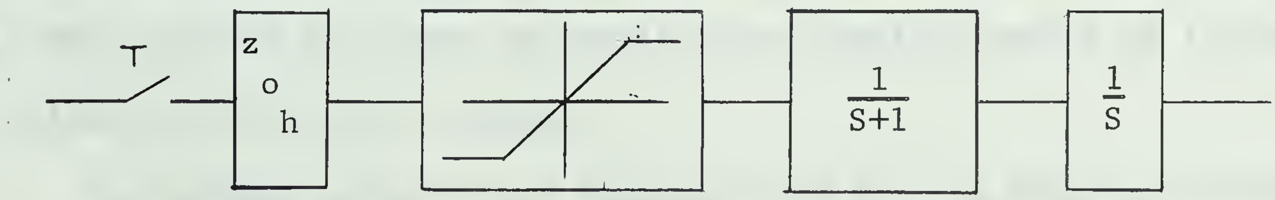


Fig. 1.3 a

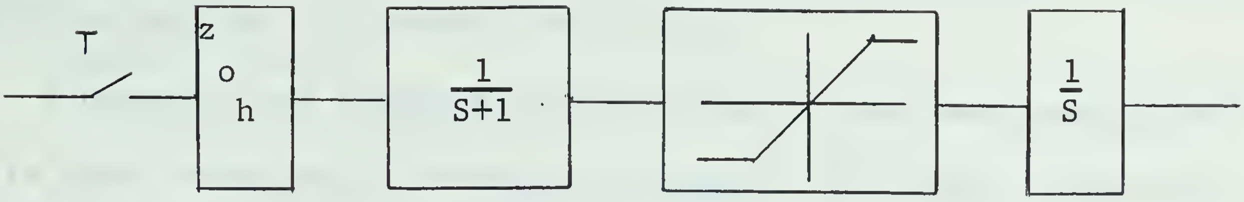


Fig. 1.3 b

The dead-beat controller designed for ramp disturbances was modified by trial and error to give a good response to step disturbances also. The non-linear systems were studied for all possible types of internal and external disturbances. The response was good in most of the cases. The effect of change of saturation limit of the non-linearity on the output was also studied.

The system used for hysteresis type of non-linearity was $\frac{1}{(S+1)}$.

The simulation of hysteresis non-linearity on the analog computer was carried out as shown in appendix B. The hysteresis non-linearity was placed in the branch of the sensing device.

Chapter IV deals with a second order oscillatory system with a transfer function of the form $\frac{\omega_n^2}{S^2+2\zeta\omega_n+\omega_n^2}, \zeta<1$. Normally a system like this will have overshoot and oscillations before settling down to its steady state value, but it was found that if a proper damping ratio is chosen for the model, an optimum response can be obtained. It was observed here that the smaller the sampling period of the controller the better the response.

The optimum value of the damping ratio for the model was chosen by trial on the analog computer. The overshoot of an oscillatory system having a damping ratio of 0.5 could be reduced to almost zero

percent with T = 0.2 sec. For a damping ratio of 0.3, the overshoot could be reduced to 3% with T = 0.2 sec.

A conventional feedback system having a dead-beat controller in it is very sensitive to parameter changes in the plant. The conditional feedback system, optimised for the output, and the corresponding conventional feedback system with dead-beat controller were compared for the same changes in parameters. It was found that the performance of the conditional feedback system was far superior to that of the conventional feedback system.

This chapter deals with linear sampled data systems.

2.1 TYPE ZERO SYSTEM

A plant with the transfer function $\frac{K_1}{(S + 1)}$ was chosen. Its response was modified by the transfer function $\frac{K_2}{(S + 2)}$ as shown in fig. 2.1.

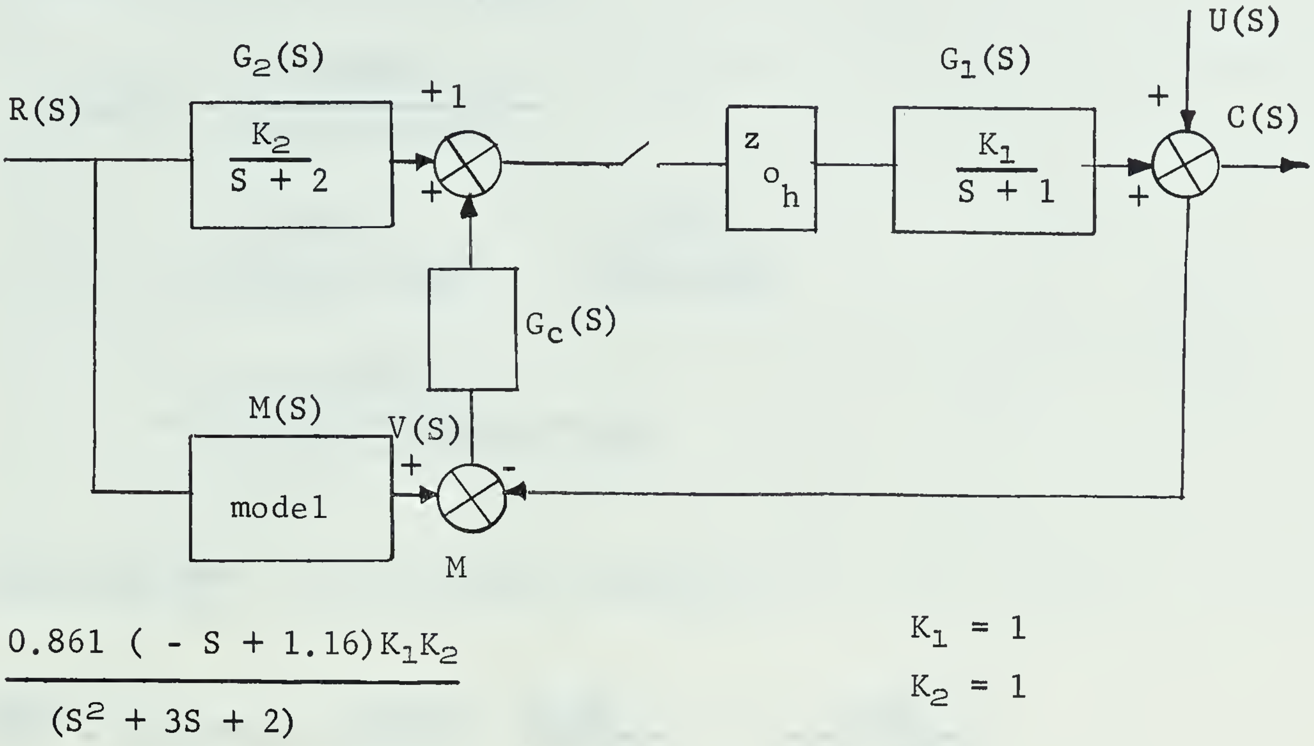


Fig. 2.1

H(S) was assumed to be unity. The model can be derived as follows:

$$G_1(S) = \frac{K_1}{S+1}$$

$$C(S) = Gho(S)G_1(S)\left\{R(S)G_2(S)\right\}^{*\mp}$$

$$C^*(S) = \left\{Gho(S)G_1(S)\right\}^*\left\{R(S)G_2(S)\right\}^*$$

$$C(Z) = GhoG_1(Z) \times RG_2(Z)$$

T = 1 sec.

∓ * Shows the sampled signals

Now $Gho(S)G_1(S) = \dfrac{1 - e^{-TS}}{S} \times \dfrac{K_1}{(S+1)}$

and $GhoG_1(Z) = \dfrac{0.632K_1}{(Z - 0.368)}$

$$R(S) \times G_2(S) = \frac{1}{S} \times \frac{K_2}{S(S+2)}$$

$$RG_2(Z) = \frac{0.432K_2Z}{(Z-1)(Z-0.136)}$$

$$C(Z) = \frac{0.432K_2Z}{(Z-1)(Z-0.136)} \times \frac{0.632K_1}{(Z-0.368)}$$

$$= \frac{0.2725K_1K_2Z}{(Z-1)(Z-0.136)(Z-0.368)}$$

Splitting $\dfrac{C(Z)}{Z}$ into partial fractions

$$\frac{C(Z)}{Z} = 0.2725K_1K_2\left\{\frac{1.833}{(Z-1)} + \frac{4.98}{(Z-0.136)} - \frac{6.82}{(Z-0.368)}\right\}$$

Taking the inverse Z- transform:

$$C(S) = 0.2725K_1K_2\left\{\frac{1.833}{S} + \frac{4.98}{S+2} - \frac{6.82}{S+1}\right\}$$

$$= \frac{0.861K_1K_2(-S+1.16)}{S(S+1)(S+2)}$$

This must also be the Laplace transform of the output of the model

$$R(S)xM(S) = \frac{0.861K_1K_2(-S+1.16)}{S(S^2+3S+2)}$$

$$R(S) = \frac{1}{S}$$

$$M(S) = \frac{0.861K_1K_2(-S+1.16)}{(S^2+3S+2)}$$

The signal flow graph for the model is shown in fig. 2.2.

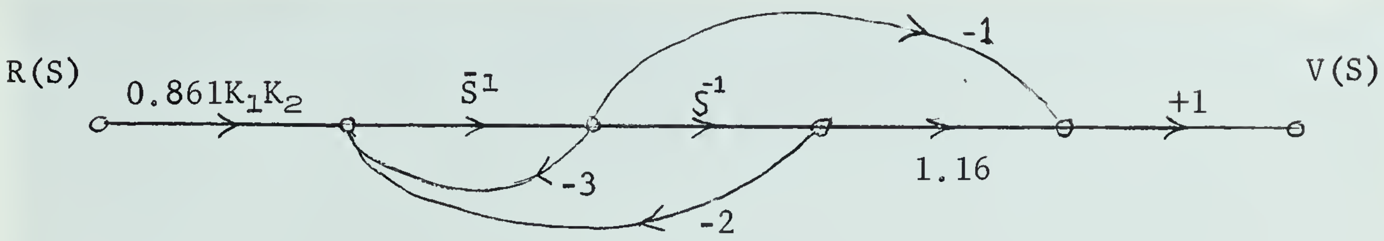
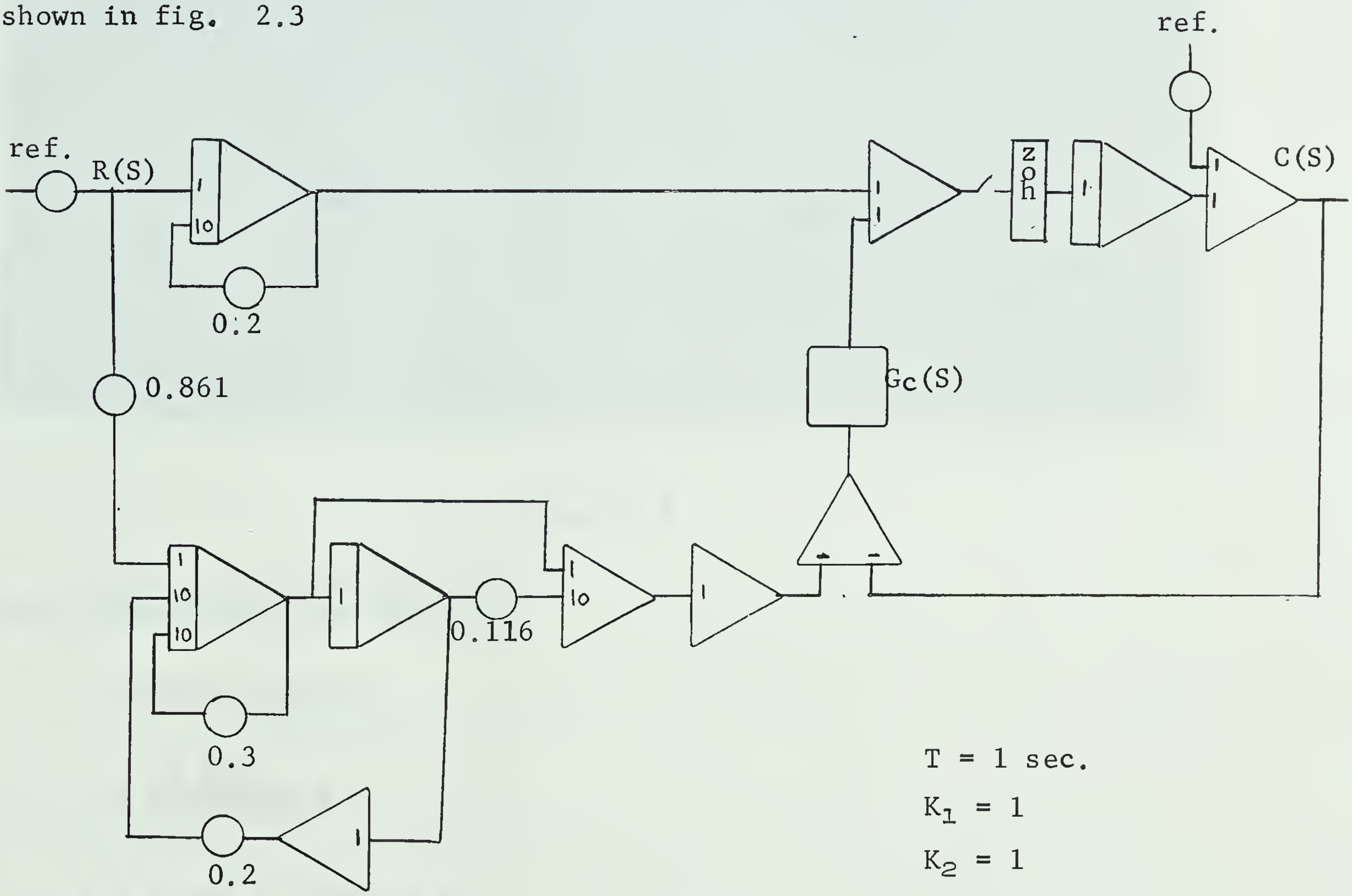


Fig. 2.2

The analog computer diagram of the conditional feedback arrangement is shown in fig. 2.3



T = 1 sec.

$K_1 = 1$

$K_2 = 1$

Fig. 2.3

The response of the system to a step input is shown in fig. 2.4. This is with $G_c(S)$ as 1. $G_c(S)$ was then assumed to be K and Routh-Hurwitz criterion applied to study the stability of the feedback loop.

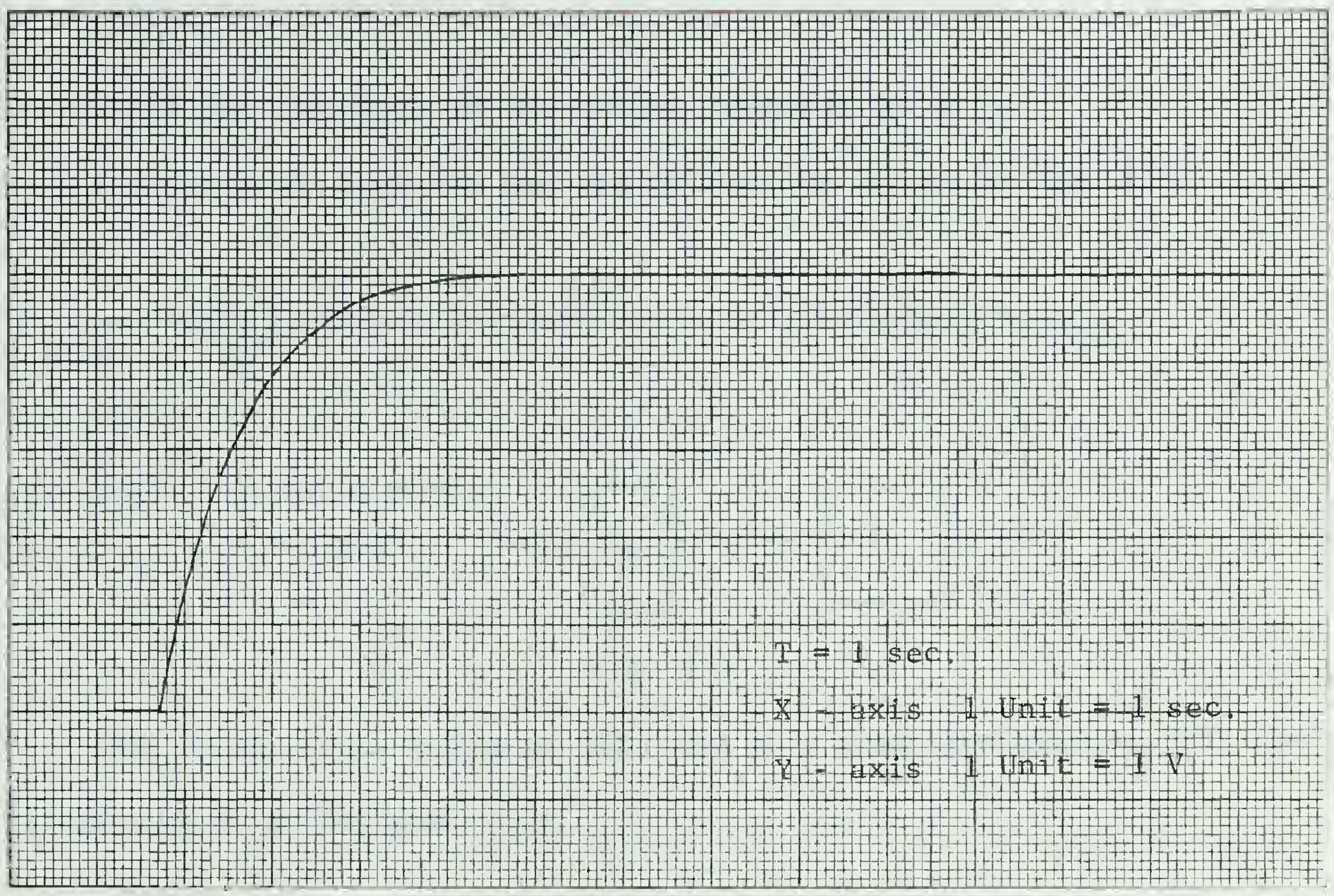


Fig. 2.4.

The characteristic equation is

$$1 + KGhoG_1(Z) = 0$$

$$1 + \frac{0.632K}{(Z-0.368)} = 0$$

$$Z - 0.368 + 0.632K = 0$$

Applying the bi-linear transformation

$$Z = \frac{r+1}{r-1} \text{ yields,}$$

$$\frac{r+1}{r-1} - 0.368 + 0.632K = 0$$

$$\therefore \; r = \frac{0.632K - 1.368}{0.632K + 0.632}$$

The root r must lie in the left half r-plane for the system to be stable.

Hence, the system is unstable for

$K > \frac{1.368}{0.632}$ i.e. $K > 2.17$

This value was verified on the analog computer as 2.15.

The feedback loop for the disturbance is shown in fig. 2.5.

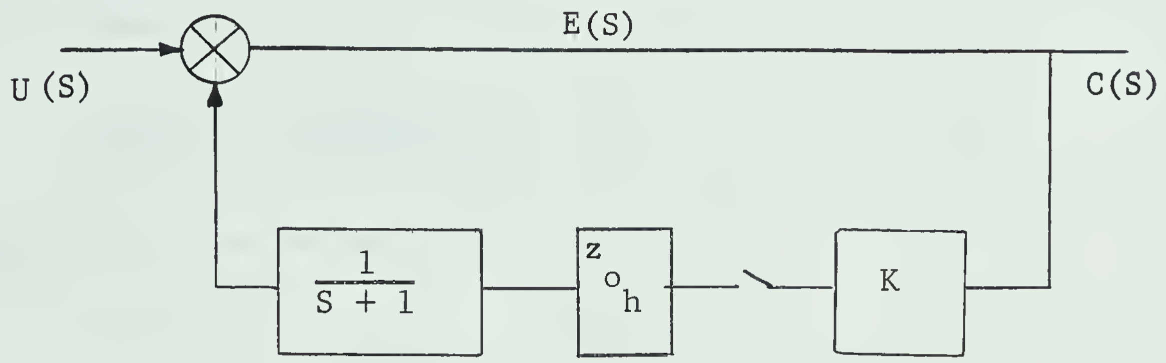


Fig. 2.5

The steady state value of C(S) due to a step disturbance U(S) can be calculated as follows:

$$C(S) = E(S)$$

$$E(S) = U(S) - KGho(S)G_1(S)C^*(S)$$

$$C^*(S) = \frac{U^*(S)}{1 + K\{Gho(S)G_1(S)\}^*}$$

$$C(Z) = \frac{U(Z)}{1 + KGhoG_1(Z)}$$

Applying the final value theorem

$$\lim_{t\to\infty} C^*(t) = \lim_{Z\to 1} (1 - Z^{-1}) \frac{U(Z)}{1 + KGhoG_1(Z)}$$

$$= \lim_{Z\to 1} \frac{(1 - Z^{-1})}{(1 - Z^{-1})\left\{1 + K\frac{(Z - 1)}{Z} \times \frac{Z}{(Z - 1)} \times \frac{0.632}{(Z - 0.368)}\right\}}$$

$$= \frac{1}{1 + K}$$

This shows that the larger the value of K, the smaller will be the steady state effect on the output due to a step disturbance.

K was taken as 5 and then a digital compensator was designed to make the system stable. The bi-linear transformation method [2] was used as follows:

$$GhoG_1(Z) = \frac{0.632K}{(Z - 0.368)}$$

Applying W transformation

$$Z = \frac{1 + W}{1 - W}$$

$$GhoG_1(W) = \frac{0.632K}{\frac{1 + W}{1 - W} = 0.368}$$

$$= \frac{0.632(1 - W)K}{0.632 + 1.368W}$$

$$= \frac{K\ (1 - W)}{1 + 2.17W}$$

$$GhoG_1(j\omega_W) = \frac{K(1 - j\omega_W)}{1 + 2.17j\omega_W}$$

The Bode diagram for $\frac{1(1 - \omega j_W)}{(1 + 2.17j\omega_W)}$ plotted in fig. 2.6. Phase lag compensation was chosen. Phase lead compensation increases bandwidth and is also not so effective if the phase lag increases rapidly in the vicinity of the gain cross over. The increase in the bandwidth is not desirable in sampled data systems. The sampling switch produces harmonics at its output. If the bandwidth is increased, the harmonics will be less suppressed and the response will not be so smooth.

The new gain cross-over was chosen as 0.6. The uncompensated system has a gain of 11.5 db at this frequency.

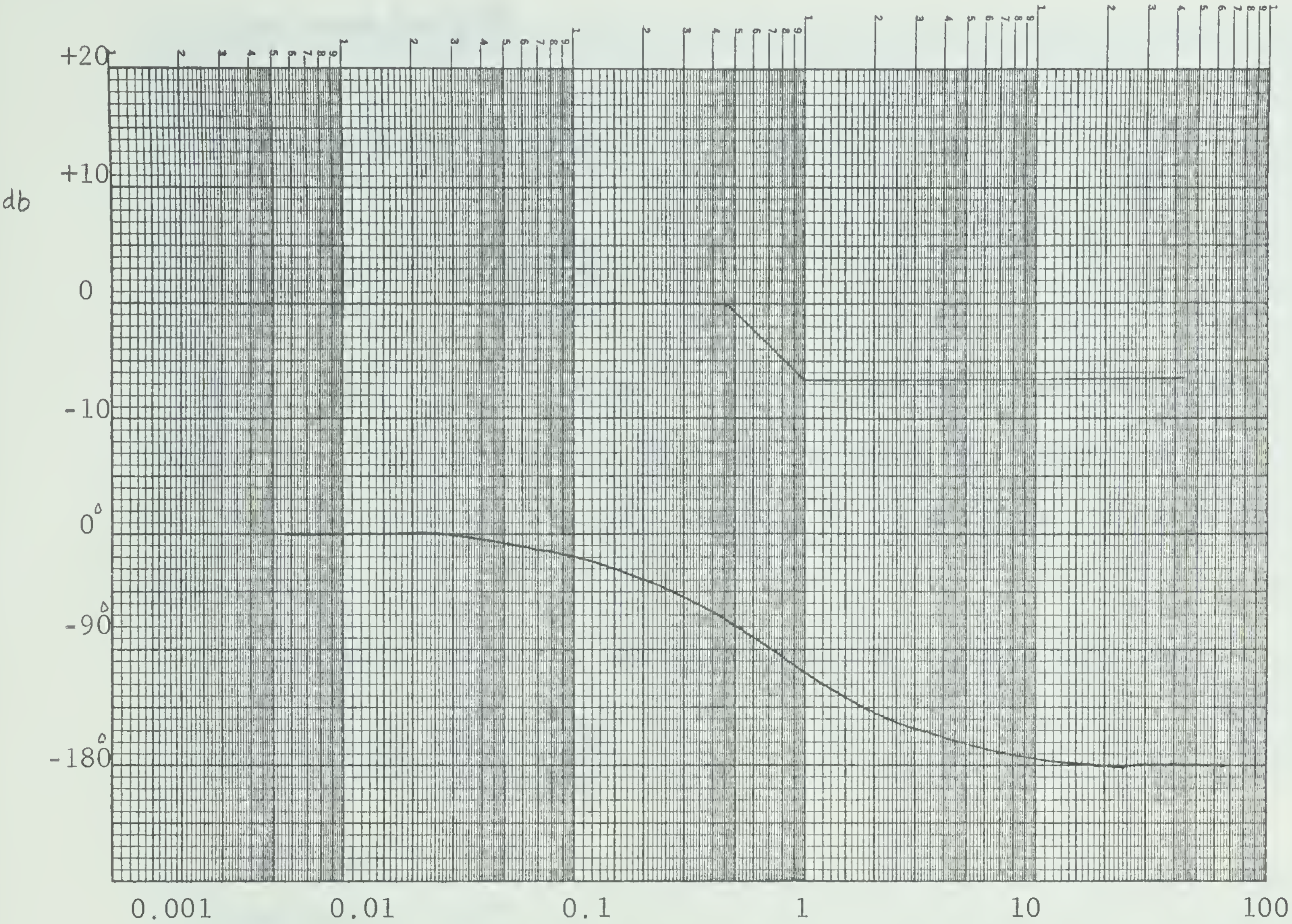


Fig. 2.6

The phase lag compensator is of the form

$$G_c(W) = \frac{1 + a\tau W}{1 + W}$$

Then $20 \log a = -11.5$

$$\log \frac{1}{a} = 0.575$$

$$a = 0.266$$

Placing the upper corner frequency one decade below the new cross-over frequency.

$$\frac{1}{a\tau} = 0.06$$

$$\tau = 62.65$$

$$G_c(W) = \frac{1 + 16.7W}{1 + 62.65W}$$

putting $W = \frac{Z - 1}{Z + 1}$ yields,

$$G_c(Z) = 0.278 \frac{1 - 0.887Z^{-1}}{1 - 0.968Z^{-1}}$$

Gain margin of this system was 4 db and the phase margin 100°. Response to a step disturbance with this controller is shown in fig. 2.7 - 1*. The settling time was observed to be too long with this compensator.

The new gain cross-over was then chosen as 0.8 and D(Z) was

$$D(Z) = 0.333 \frac{1 - 0.8525Z^{-1}}{1 - 0.952Z^{-1}}$$

Gain margin = 2 db

Phase margin = 81°

The response to a step disturbance is shown in fig. 2.7 - 2.

Next K was chosen as 10 and with the new gain cross-over as 0.8.

$$D(Z) = 0.169 \frac{1 - 0.8525Z^{-1}}{1 - 0.975Z^{-1}}$$

Gain margin = 2 db

Phase margin = 81°

A free integrator was then tried as the compensator. Analytically the effect of a step disturbance on the output with this compensator should be zero in the steady state. The system was tried on the analog computer and it showed a much too oscillatory response to a step disturbance.

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* dash shows the curve number on that figure.

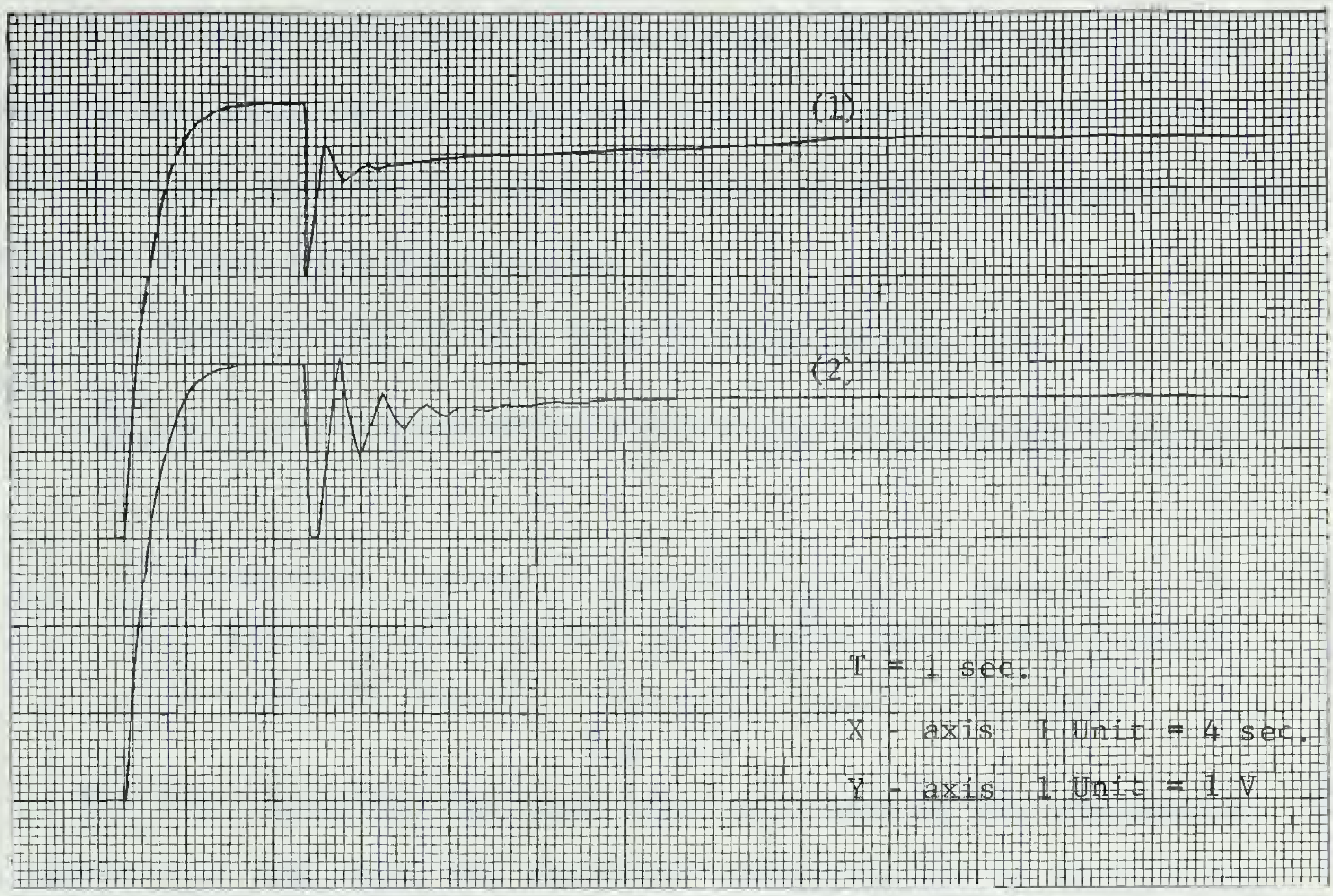

(1)
(2)
T = 1 sec.
X - axis 1 Unit = 4 sec.
Y - axis 1 Unit = 1 V

Fig. 2.7

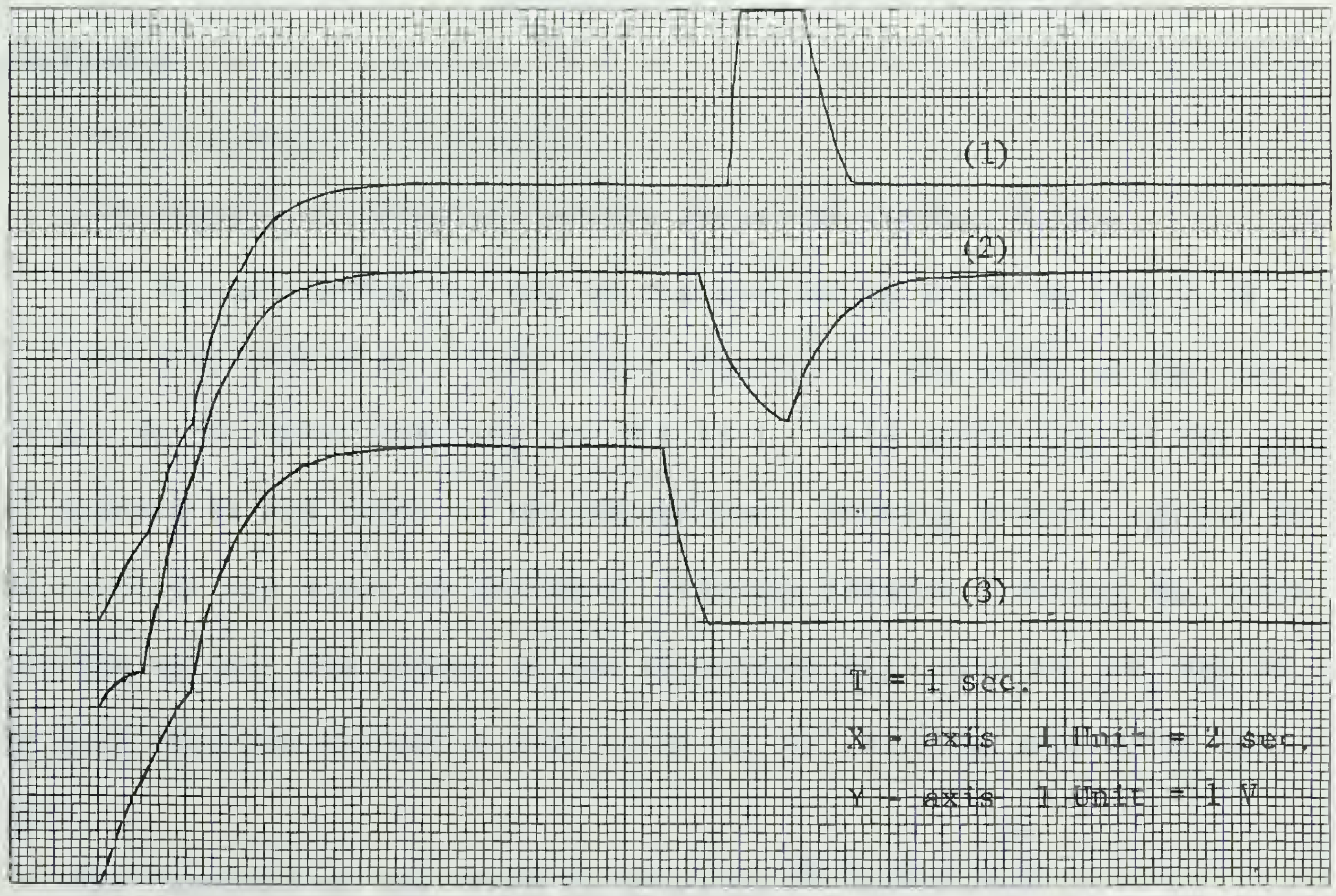

(1)
(2)
(3)
T = 1 sec.
X - axis 1 Unit = 2 sec.
Y - axis 1 Unit = 1 V

Fig. 2.8

A dead-beat controller was then tried. This is a type zero system and therefore an integrator has to be used with the controller, so that when the input to the controller goes to zero the integrator should hold the output value(see appendix A).

The feedback loop in this case is shown in fig. 2.9.

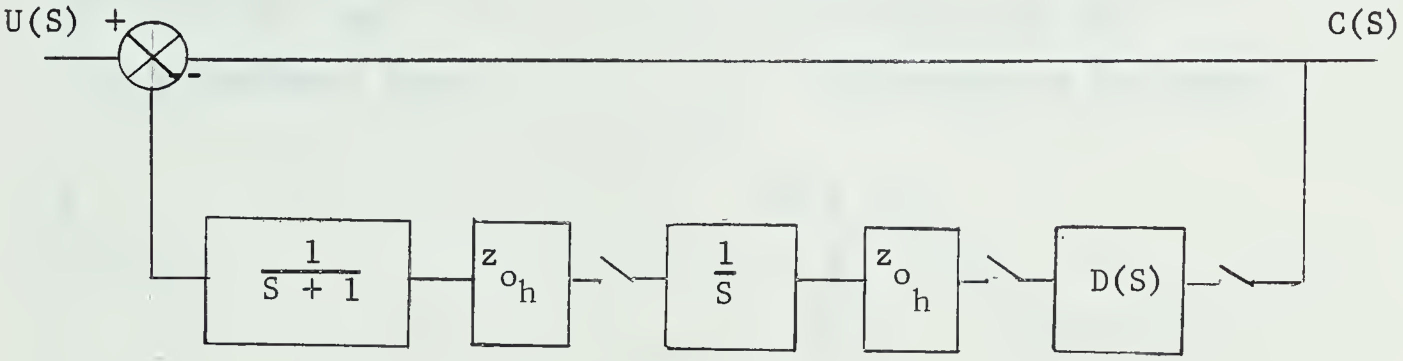


Fig 2.9

The free integrator has been added as a part of the compensator. In a simple case like this, the dead-beat controller can be derived step by step as shown below. A general transfer function $\frac{K}{(S + C)}$ is considered.

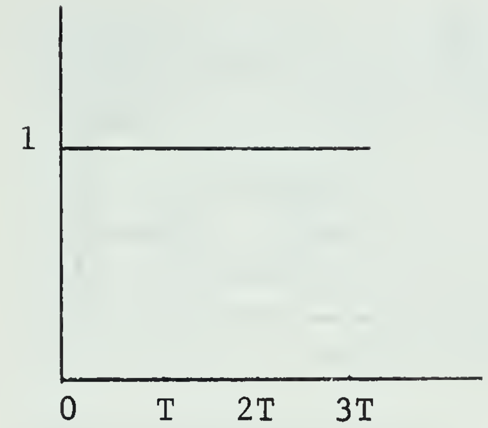
Sampling period = T sec.

The signals at different points are shown in fig. 2.10.

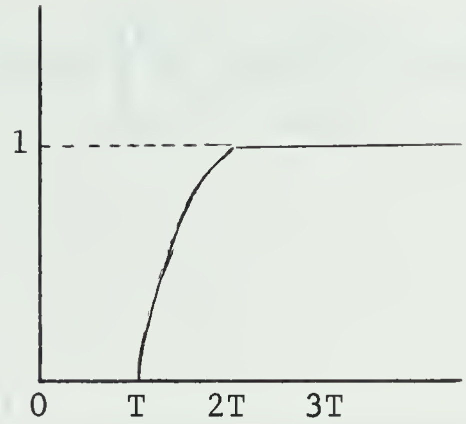
The digital controller can be represented as shown in fig. 2.11

The disturbance is a unit step. Therefore, the input to the controller at the first sampling instant is 1. This multiplied by the gain 'a' appears at the input to the integrator. Nothing will be applied at the input to the plant at the first sampling instant. During the first sampling period, the output at the integrator will rise to 'aT' and at the next sampling instant a step input of 'aT' will be applied to the plant.

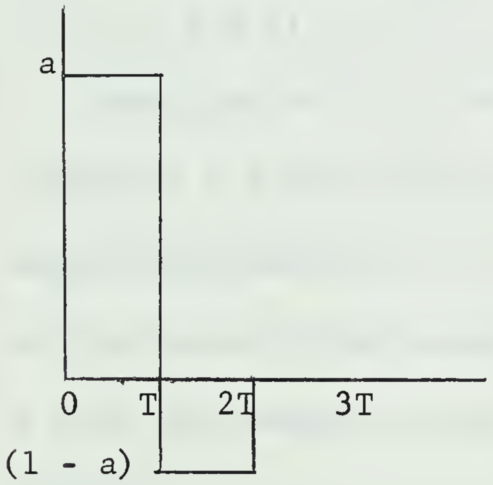
The response of the plant is given by $\frac{aT\,K\,(1 - e^{-Ct})}{C}$



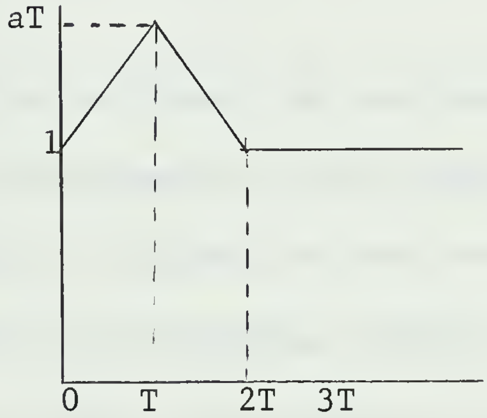
(a) Disturbance input



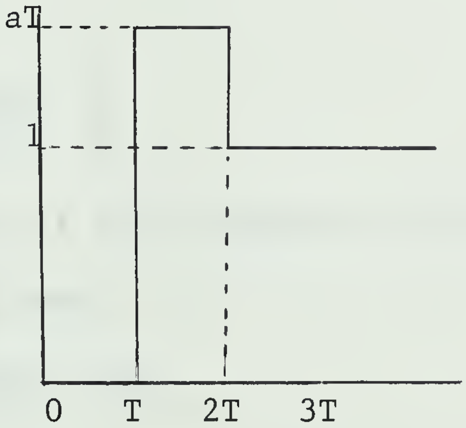
(b) Output of the plant



(c) Input to the integrater



(d) Output of the integrater



(e) Input to the plant

Fig. 2.10

For dead-beat response, this should be unity at the end of one sampling period

$$\frac{aT\,K\,(1 - e^{-CT})}{C} = 1$$

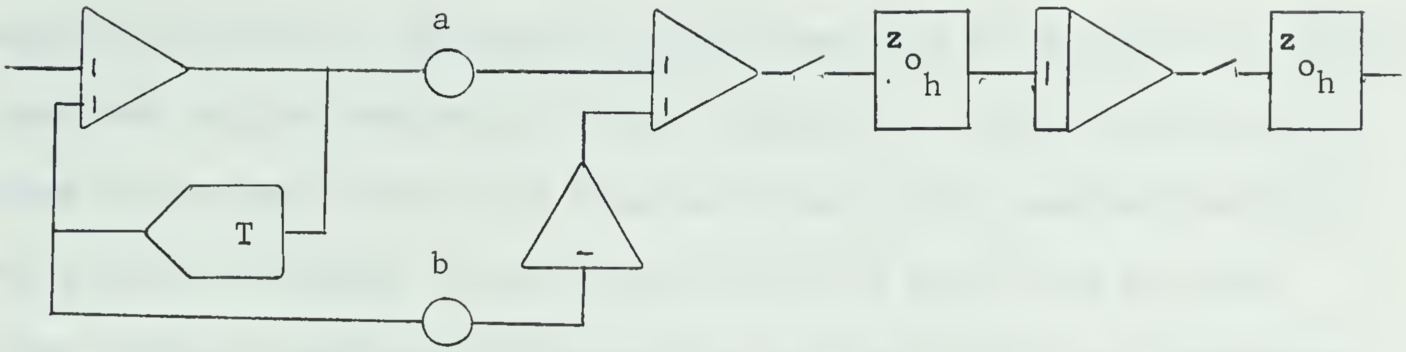


Fig. 2.11

$$a = \frac{C}{T\,K\,(1 - e^{-CT})}$$

Next the value of the gain 'b' is to be found. At the sampling instant t = T the negative feedback of the time delay will cancel the unit disturbance input to the controller. A signal has now to be applied to the input of the integrator, so that at the sampling instant t = 2T, the output of the integrator is unity. The value of 'b' will be

$$\frac{(1 - a)}{T}$$

$$\text{or } b = \left\{1 - \frac{C}{TK(1 - e^{-CT})}\right\} \frac{1}{T}$$

After that the output of the integrator will stay at unity and the input to the controller at zero.

In this particular case

K = 1, T = 1, C = 1

a = 1.582, b = -0.582

$$D(Z) = \frac{1.582 - 0.582Z^{-1}}{1 + Z^{-1}}$$

The response of the system to a step input with this dead-beat controller as the compensator is shown in fig. 2.8 - 1. A step distur-

bance was applied at the output of the plant when the response to the step input had settled down to its steady state value. The output of the plant was brought back to its original value in two sampling periods. The process of removal of the disturbance will start from the sampling instant immediately following the application of the disturbance. Thus the actual time of removal of the disturbance can be anywhere between two and three sampling periods. This additional time Ta is given by the following inequality

$$0 < Ta < T$$

This comment about the additional time will hold for all other cases to be studied.

The step disturbance was then applied at the point '1' in fig. 2.1 and the response is shown in fig. 2.8 - 2. This disturbance may also be considered as a variation in the parameter or the gain of the transfer function $G_2(S)$, because that will have a similar effect at the point '1'.

The response to a step disturbance at 'm' in fig. 2.1 is shown in fig. 2.8 - 3. It may be noted here that a disturbance at 'm' gives rise to an equal disturbance at the output, hence care will have to be taken that the transfer function H(S) is not subject to large disturbances.

An analog integrator in the controller is undesirable, because if there is any drift, it will start building up at the integrator. Moreover, the use of the analog integrator requires an additional sampler and a zero order hold. The zero order hold and the analog integrator may be combined with the controller. The Z-transform of a zero order hold and an integrator is $\frac{Z^{-1}}{(1 - Z^{-1})}$. The digital integrator is shown in fig. 2.12.

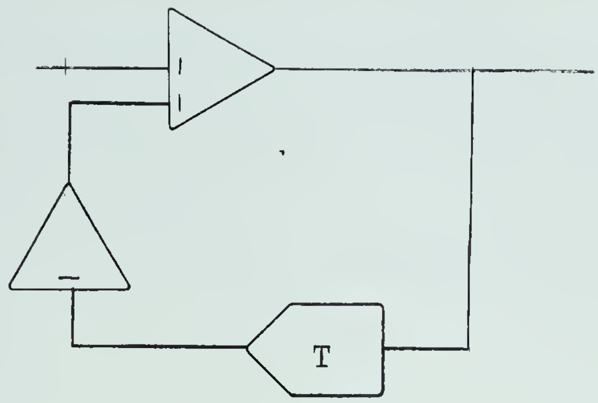


Fig. 2.12 Digital integrater

$$\text{Modified } D(Z) = \frac{(1.582 - 0.582Z^{-1})}{(1 + Z^{-1})} \times \frac{Z^{-1}}{(1 - Z^{-1})}$$

$$D(Z)_m = \frac{1.582Z^{-1} - 0.582Z^{-2}}{(1 - Z^{-2})}$$

This was tried as shown in fig. 2.13 - 1. It worked well.

Internal variations in the plant such as gain variations and time constant variations were studied. For the transfer function $\frac{K_1}{(S + 1)}$, an instantaneous gain variation or a time constant variation, will be equivalent to the sum of step and exponential disturbances at the output of the plant. The dead-beat controller designed for step disturbances was found to behave well for these combined disturbances also. The results are shown in table 2.1

These changes in the parameter were affected instantaneously by switches. In actual practice the change is likely to occur gradually and therefore the output will not be disturbed too much.

The unevenness in the transient response is due to the fact the input to the plant and the model is not applied at the same moment. Signal to the plant is applied at the sampling instant that follows the

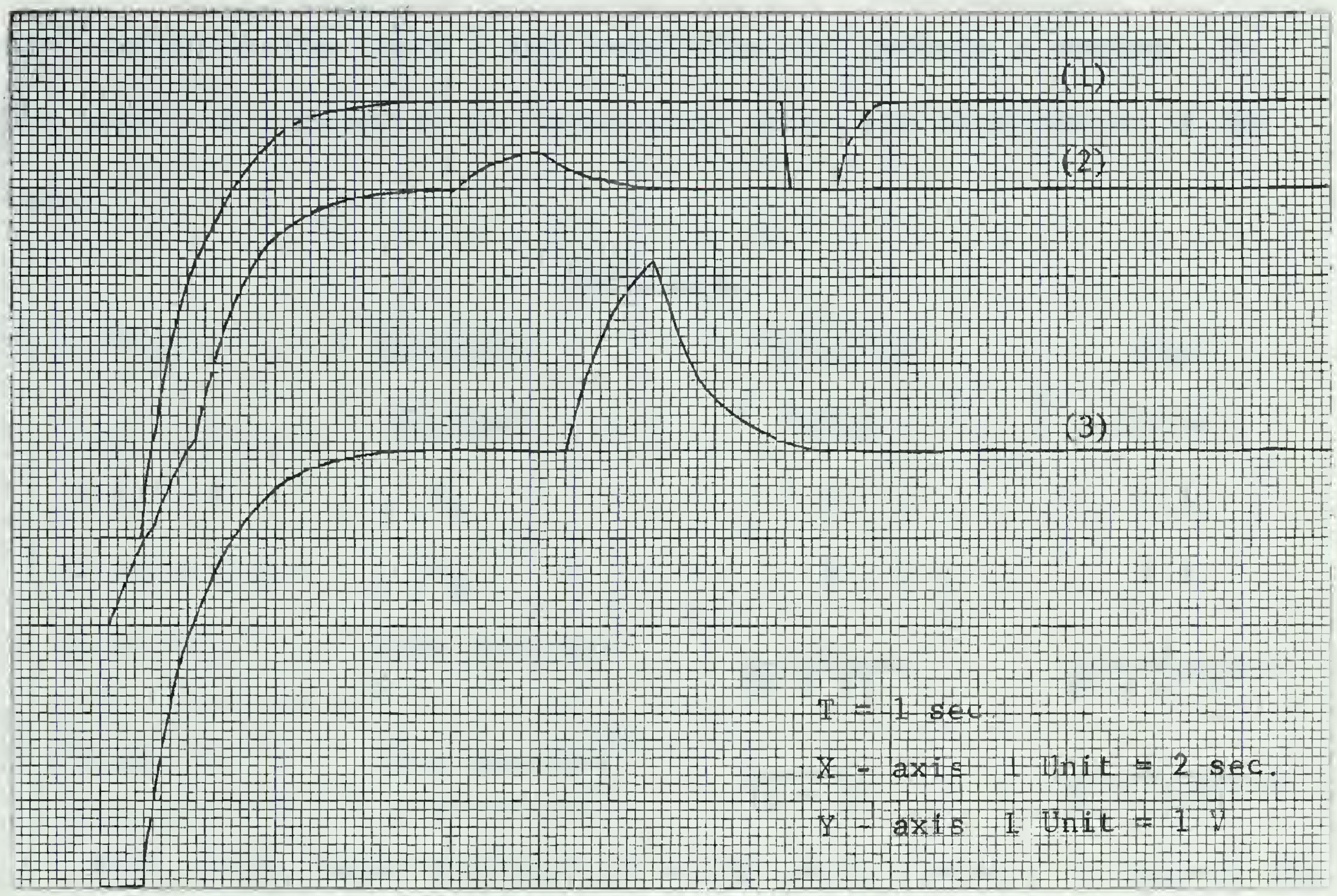


Fig. 2.13

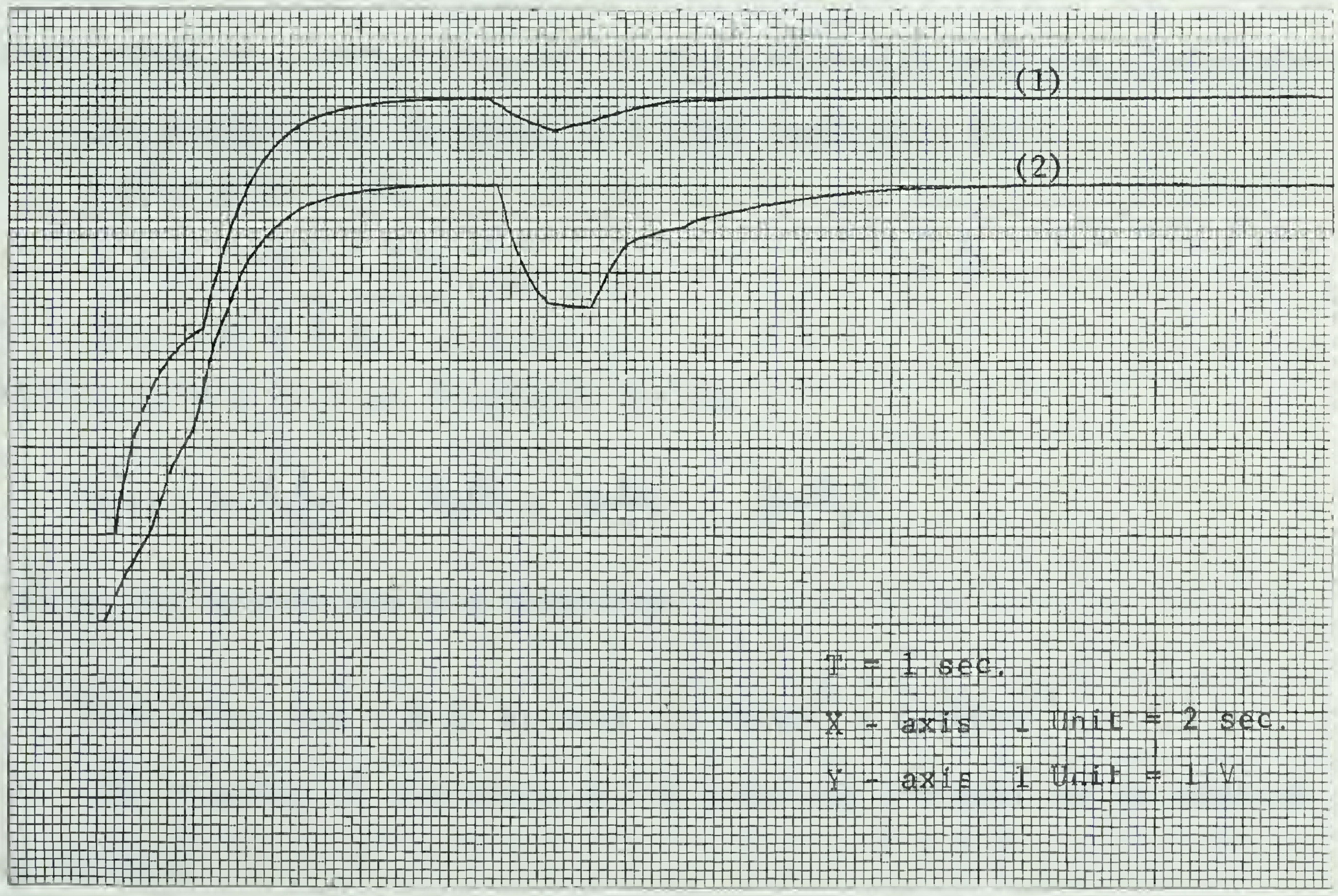


Fig. 2.14

application of the input. The model does not contain a sampler, so the signal is applied to it immediately after the application of the input.

TABLE 2.1

Fig.	Parameter Changed	Original Value	New Value
2.13 - 2	K_2	1	1.1
2.13 - 3	K_2	1	1.5
2.14 - 1	Time constant	$\frac{1}{(S + 1)}$	$\frac{1}{(S + 1.1)}$
2.14 - 2	Time constant	$\frac{1}{(S + 1)}$	$\frac{1}{(S + 1.5)}$

Next a ramp input was applied to the above system. It removed the external disturbances as before. This is shown in fig. 2.15. The application of a ramp input can be analyzed analytically as shown below.

The response of the model to a ramp input will be

$$V(S) = R(S) \times M(S)$$

$$\frac{0.861K_1\ (-S + 1.16)}{S^2(S + 1)(S + 2)}\ ,\ K_2 = 1$$

Splitting into partial fractions yields,

$$V(S) = K_1\left(\frac{1}{2S^2} - \frac{0.25}{S} + \frac{1.86}{S + 1} - \frac{0.68}{S + 2}\right)$$

Taking the inverse Laplace transform

$$V(t) = 0.5\ K_1t - 0.25\ K_1\ U(t) + 1.86K_1e^{-t} - 0.68K_1e^{-2t}$$

The response of the plant at the sampling instants can be derived as follows:

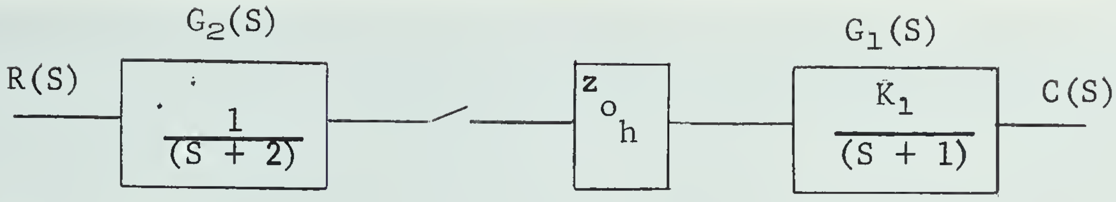


Fig 2.15 (a)

The open loop system is shown in fig. 2.15 (a).

$$C(S) = \left\{R(S) \times \frac{1}{S+2}\right\}^{*} \times (1 - e^{-TS}) \times \frac{K_1}{S(S+1)}$$

$$C^{*}(S) = \left\{R(S) \times \frac{1}{S+2}\right\}^{*} \times \left\{(1 - e^{-TS}) \frac{K_1}{S(S+1)}\right\}^{*}$$

$$C(Z) = RG_2(Z) \times (1 - Z^{-1})\, \mathcal{Z}\left\{\frac{K_1}{S(S+1)}\right\}$$

$$RG_2(Z) = \frac{0.5Z(0.568Z + 0.296)}{(Z-1)^2(Z-0.136)}$$

$$C(Z) = \frac{0.5Z(0.568Z + 0.296)}{(Z-1)^2(Z-0.136)} \times \frac{0.632K_1}{(Z-0.368)}$$

Dividing $\frac{C(Z)}{Z}$ into partial fractions

$$\frac{C(Z)}{Z} = 0.316K_1\left\{\frac{1.585}{(Z-1)^2} - \frac{3.3}{Z-1} + \frac{5.45}{Z-0.368} - \frac{2.155}{Z-0.136}\right\}$$

Taking the inverse Z-transform

$$C^{*}(t) = 0.5K_1 t - 1.04K_1 U(t) + 1.72K_1 e^{-t} - 0.68K_1 e^{-2t}$$

Comparing the expressions for $C^{*}(t)$ and $C(t)$, it is found that the two will differ by a step signal in the steady state. This comparison is at the sampling instants only, but if there is no inter-sample ripple, the comparison will hold at all the points of the output. This step signal will be treated as a disturbance in the feedback loop and hence

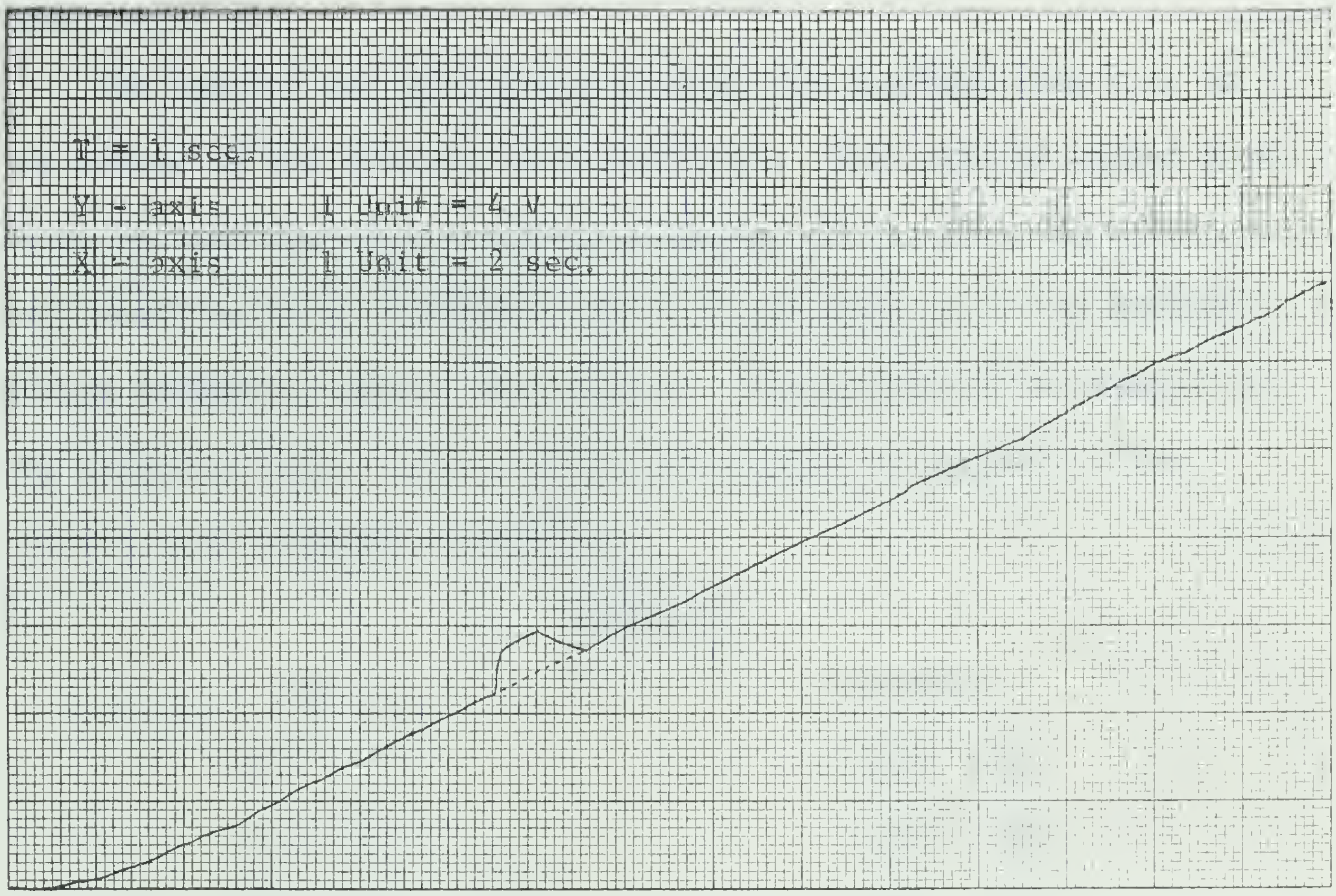


Fig. 2.15

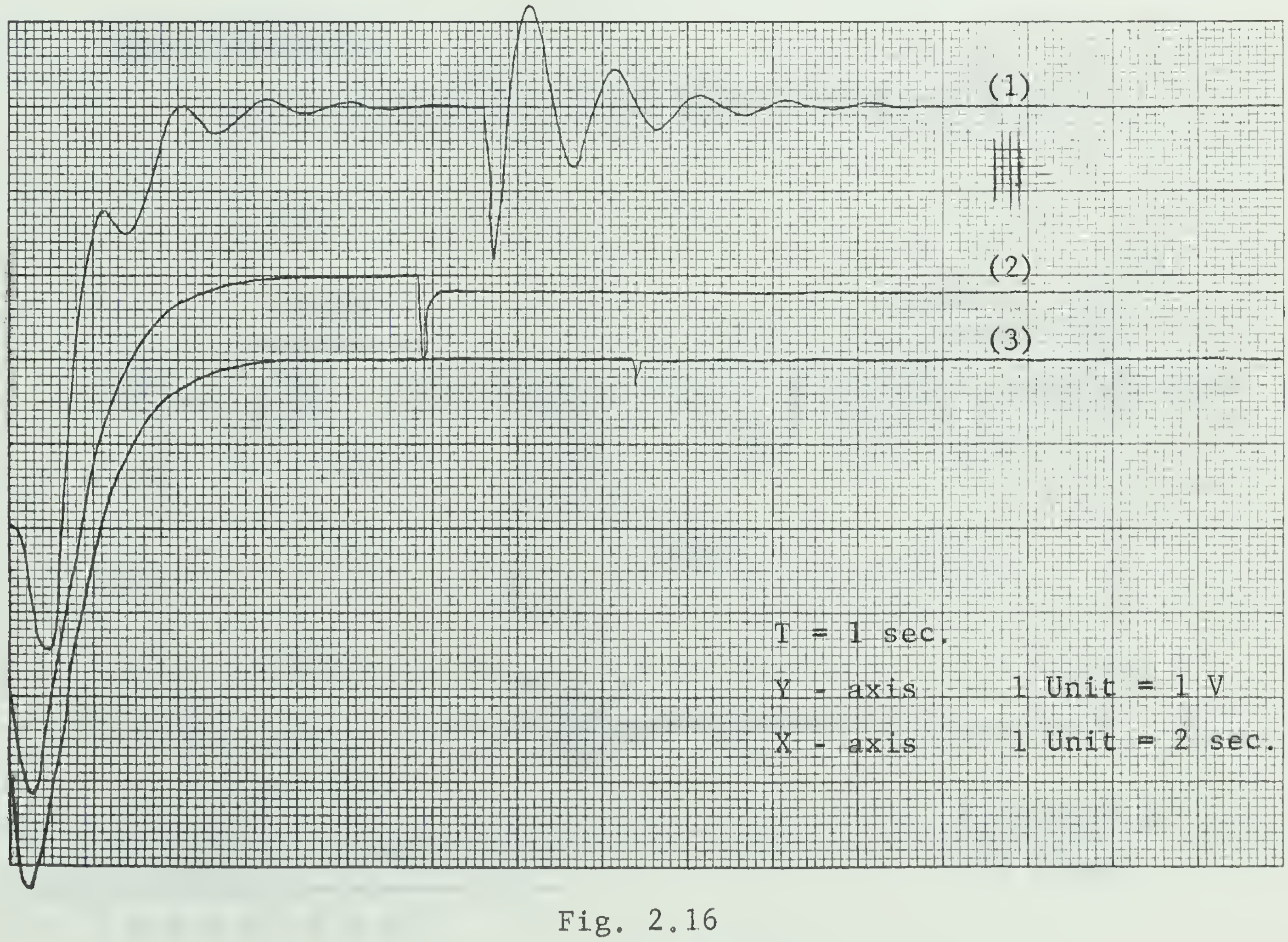


Fig. 2.16

compensated for.

The sampler was then placed outside the feedback loop i.e. in the branch of the transfer function $G_2(S)$. This will be useful when it is desired that the sampling period should not have any effect on the stability of the feedback loop and also when it is not desired to use expensive digital controllers. The placing of the sampler in the branch of $G_2(S)$ will, however, not always be possible. The outputs were as shown in table 2.2.

TABLE 2.2

Fig.	Remarks
2.16 - 1	$G_c(S) = \frac{1}{S}$
2.16 - 2	$G_c(S) = 10$
2.16 - 3	$Gc(S) = 100$

2.2 TYPE ONE SYSTEM

The system studied in this case is shown in fig. 2.17.

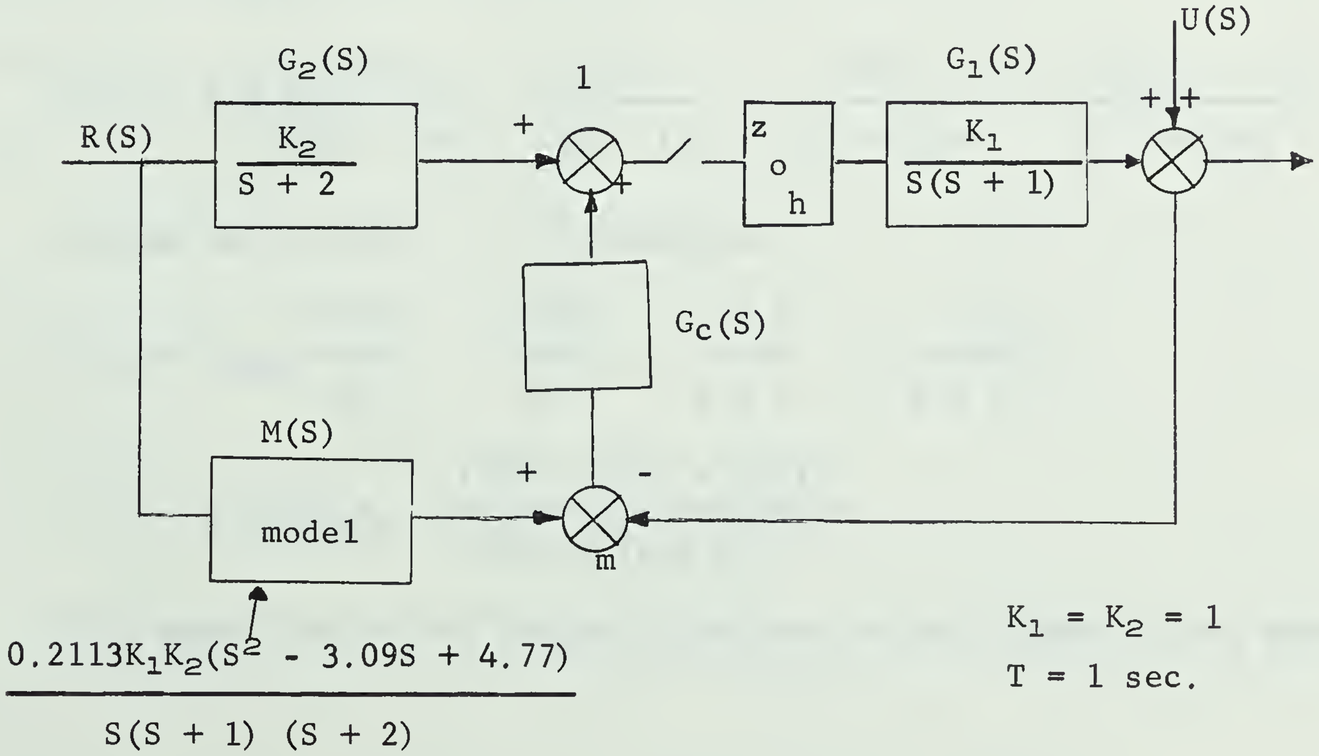


Fig. 2.17

The model can be derived as follows

$$C(S) = \left\{R(S)G_2(S)\right\} * \left\{Gho(S)G_1(S)\right\}$$

$$C^*(S) = \left\{R(S)G_2(S)\right\} * \left\{Gho(S)G_1(S)\right\} *$$

$$C(Z) = RG_2(Z) \times GhoG_1(Z)$$

$$Gho(S)G_1(S) = \frac{1 - e^{-TS}}{S} \times \frac{K_1}{S(S + 1)}$$

$$GhoG_1(Z) = \frac{0.368K_1(Z + 0.713)}{(Z - 1)(Z - 0.368)}$$

$$R(S)G_2(S) = \frac{K_2}{S(S + 2)}$$

$$RG_2(Z) = \frac{0.432K_2Z}{(Z - 1)(Z - 0.136)}$$

$$C(Z) = \frac{0.159K_1K_2\, Z\, (Z + 0.713)}{(Z - 1)^2(Z - 0.136)(Z - 0.368)}$$

Dividing $\frac{C(Z)}{Z}$ into partial fractions

$$C(Z) = K_1K_2\left\{\frac{3.14}{(Z - 1)^2} - \frac{6.77}{(Z - 1)} - \frac{4.9}{(Z - 0.136)} + \frac{11.7}{(Z - 0.368)}\right\}$$

Taking the inverse Z- transform

$$C(S) = K_1K_2\left\{\frac{3.14}{S^2} - \frac{6.77}{S} - \frac{4.9}{S + 2} + \frac{11.7}{S + 1}\right\}$$

$$= 0.2113K_1K_2 \frac{(S^2 - 3.09S + 4.77)}{S^2(S + 1)(S + 2)}$$

This must also be the Laplace transform of the output of the model

$$\frac{1}{S} \times M(S) = 0.2113K_1K_2 \frac{(S^2 - 3.09S + 4.77)}{S(S + 1)(S + 2)}$$

$$\therefore M(S) = 0.2113K_1K_2 \frac{(S^2 - 3.09S + 4.77)}{S(S + 1)(S + 2)}$$

The signal flow graph of the model is shown in fig. 2.18 (a).

The computer diagram is shown in fig. 2.18 (b).

Since it is a type one system and it is operating in the open loop configuration, there will be a ramp output for a step input. This suggests that a parameter variation in the plant will be equivalent to the sum of step, exponential and ramp disturbances at the output in the transient stage. In the steady state this will be equivalent to the sum of step and ramp disturbances. Exponential disturbances will die out. In this case a suitable digital controller will be the one which gives a good response to both, the step and the ramp disturbances.

A dead-beat controller for a step disturbance will give a constant error to a ramp disturbance whereas a dead-beat controller for a ramp disturbance will give a high overshoot to a step disturbance. A compromise is, therefore, to be made between the two.

The combined synthesis and minimal response method[2] was used to find the controller. This method was found to be quite convenient for a case like this. This method allows one to design the digital controller according to the desired maximum overshoot and the settling time.
The maximum overshoot and the settling time are related to the damping ratio and the damping constant respectively. The requirements of the proper damping ratio and the damping constant can be met because one is free to choose the roots of the characteristic equation.

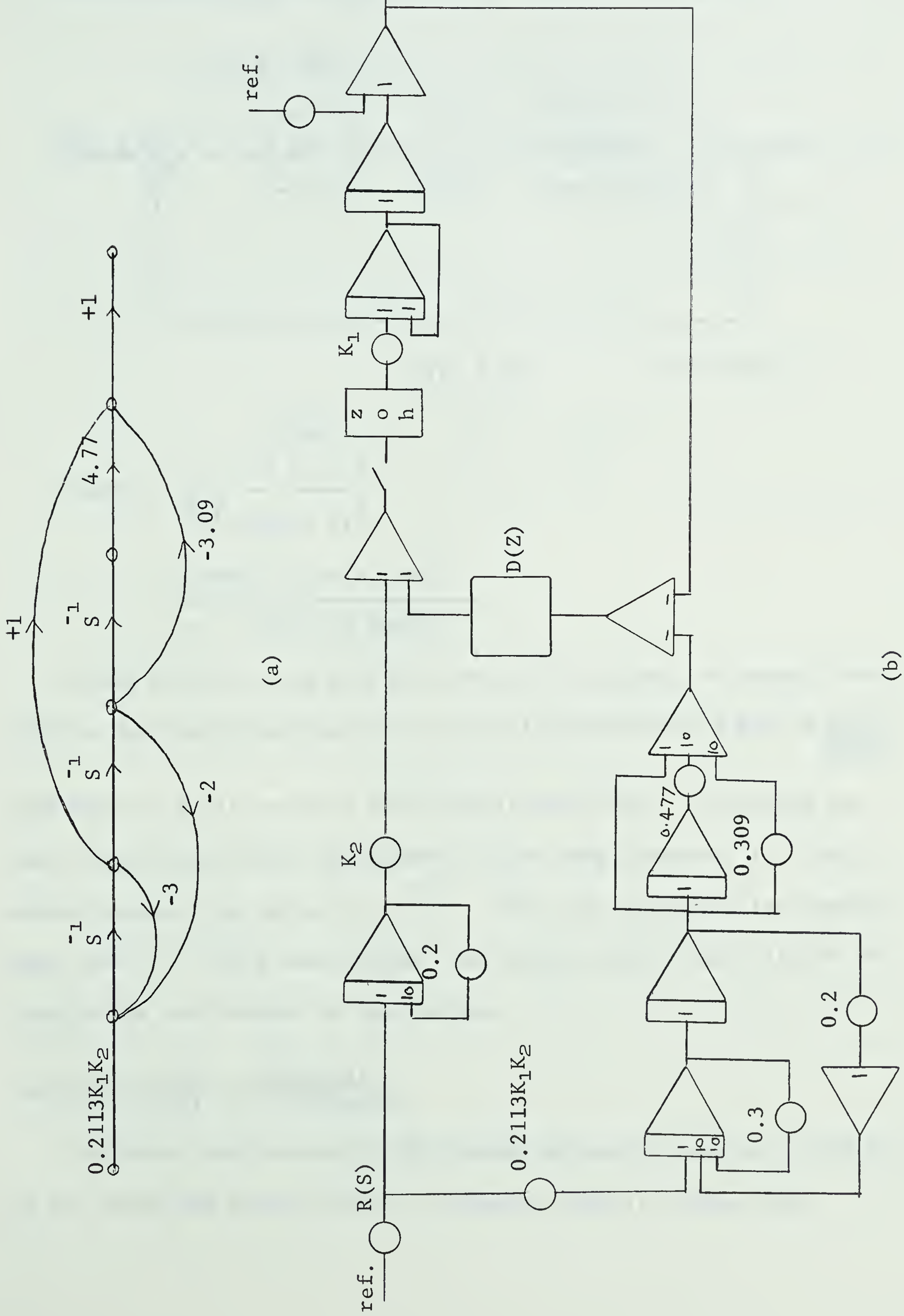

0.2113K1K2
S-1
S-1
S-1
+1
4.77
-3.09
-2
-3
+1
(a)
ref.
K1
z o h
D(Z)
K2
0.2
R(S)
ref.
0.2113K1K2
0.477
0.309
0.2
0.3
(b)

Fig. 2.18

The feedback loop can be viewed as shown in fig. 2.19.

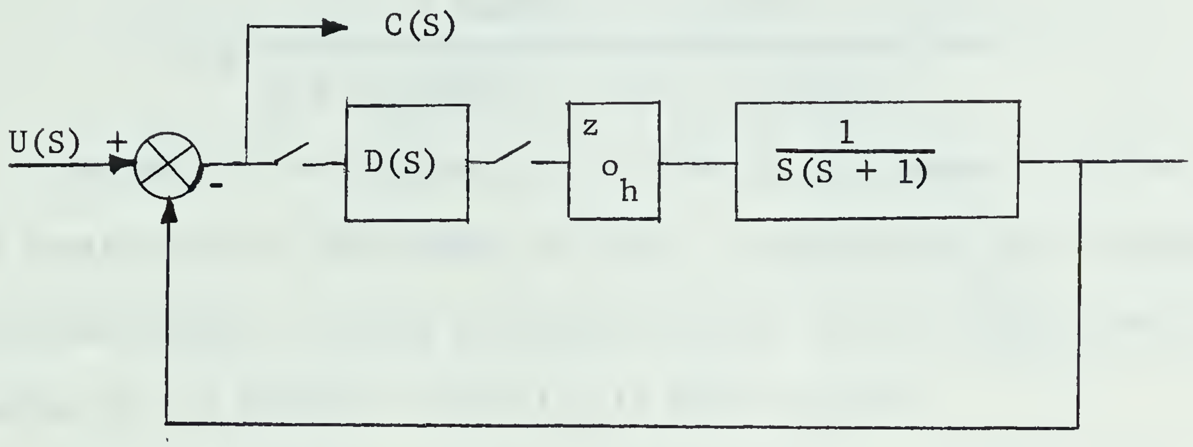


Fig. 2.19 T = 1 sec.

$$G_p(Z) = \mathcal{Z}\left\{\frac{1 - e^{-TS}}{S^2(S + 1)}\right\}$$

$$= \frac{0.368\, Z^{-1}\,(1 + 0.713Z^{-1})}{(1 - Z^{-1})(1 - 0.368Z^{-1})}$$

Since $G_p(Z)$ has one more pole than it has zeros, to insure that D(Z) is physically realizable the overall function M(Z) $\left(M(Z) = \frac{C(Z)}{R(Z)}\right)$ must have at least one more pole than it has zeros. To satisfy the zero steady state error requirement of the ramp response, (1 - M(Z)) should contain the factor $(1 - Z^{-1})^2$. This also satisfies the requirement that (1 - M(Z)) must include the poles of $G_p(Z)$ which lie on or outside the unit circle in the Z-plane.

Now $M(Z) = \frac{C(Z)}{R(Z)} = \frac{D(Z)G_p(Z)}{1 + D(Z)G_p(Z)}$

The poles and zeros of $G_p(Z)$ inside the unit circle are cancelled by the poles and zeros of D(Z). Therefore D(Z) is of the form

$$D(Z) = \frac{M(Z)}{1 - M(Z)} \times \frac{1}{Gp(Z)}$$

$$= k_d \frac{(1 - 0.368\bar{Z}^{1})(1 + a_1\bar{Z}^{1})(1 + a_2\bar{Z}^{1}) \text{ ----}}{(1 + 0.713\bar{Z}^{1})(1 - \bar{Z}^{1})(1 + b_1\bar{Z}^{1}) \text{----}}$$

For D(Z) to be physically realizable, the number of poles should be at least equal to the number of zeros. In practice, the minimum possible number of poles and zeros will be chosen so that the simulation of the digital controller is the simplest.

The open loop transfer function of the compensated system is

$$D(Z)G_p(Z) = \frac{0.368k_d\bar{Z}^{1}\ (1 + a_1\bar{Z}^{1})(1 + a_2\bar{Z}^{1}) \text{ ----}}{(1 - \bar{Z}^{1})^{2}(1 + b_1\bar{Z}^{1})(1 + b_2\bar{Z}^{1}) \text{ ----}}$$

The lowest power in Z of the system is two. In other words, the simplest form of $D(Z)G_p(Z)$ is

$$D(Z)G_p(Z) = \frac{0.368k_d\bar{Z}^{1}\ (1 + a_1\bar{Z}^{1})}{(1 - \bar{Z}^{1})^{2}}$$

which has two unknowns k_d and a_1. The characteristic equation of the system is

$$1 + \frac{0.368k_d\bar{Z}^{1}\ (1 + a_1\bar{Z}^{1})}{(1 - \bar{Z}^{1})^{2}} = 0$$

$$1 + (0.368\ k_d - 2)\bar{Z}^{1} + (1 + 0.368k_d\ a_1)\bar{Z}^{2} = 0 \qquad 2.1$$

The damping ratio curves shown in fig. 2.20 can now be used and suitable poles chosen for the system. The poles were first chosen as $0.27 \pm j\ 0.32$, to give a damping ratio of 0.707. The characteristic equation with these roots is

$$(Z - 0.27 - j\ 0.54)(Z - 0.27 + j\ 0.54) = 0$$

$$1 - 0.54\bar{Z}^{1} + 0.1754\bar{Z}^{2} = 0 \qquad 2.2$$

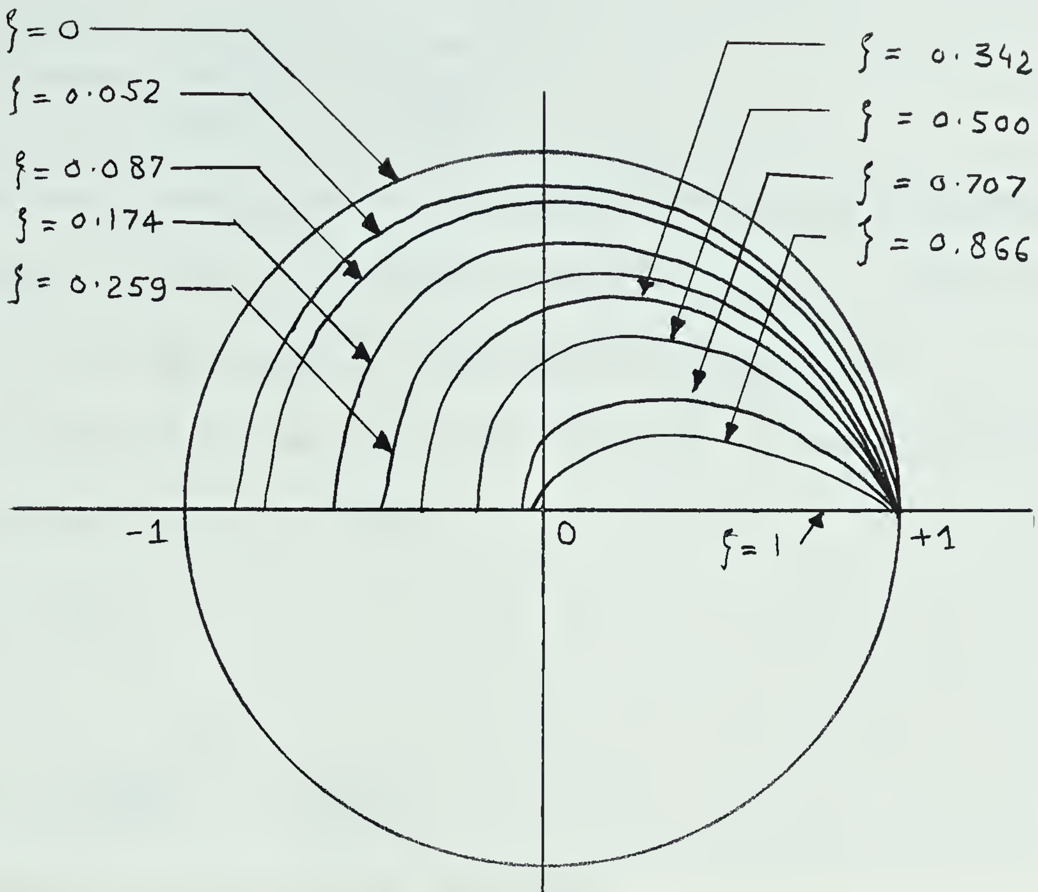


Fig. 2.20

Equating the coefficients of equal powers in equations 2.1 and 2.2.

$0.368k_d - 2 = -0.54$

$k_d = 3.967$

and $1 + 0.368k_d a_1 = 0.1754$

$a_1 = -0.563$

$$D(Z) = 3.967 \frac{(1 - 0.368Z^{-1})(1 - 0.563Z^{-1})}{(1 + 0.713Z^{-1})(1 - Z^{-1})}$$

$$= 3.967 \frac{(1 - 0.931Z^{-1} + 0.2072Z^{-2})}{(1 - 0.267Z^{-1} - 0.713Z^{-2})}$$

Then the roots were chosen as $0.5 \pm j\,0.15$ to give a damping ratio of 0.88 and the D(Z) found was

$$D(Z) = 2.72 \frac{(1 - 1.0955Z^{-1} + 0.268Z^{-2})}{(1 - 0.287Z^{-1} - 0.713Z^{-2})}$$

A dead-beat controller for ramp disturbances was then designed. The performance of these controllers was then compared. The design procedure[2] for the dead-beat controller is shown below.

Since there is a ramp input and $G_p(Z)$ has no zeros outside the unit circle, $M(Z)$ will be of the form

$$M(Z) = 2Z^{-1} - Z^{-2}$$

$$D(Z) = \frac{M(Z)}{1 - M(Z)} \times \frac{1}{G_p(Z)}$$

Substituting the values of $M(Z)$ and $G_p(Z)$

$$D(Z) = \frac{(2Z^{-1} - Z^{-2})(1 - Z^{-1})(1 - 0.368Z^{-1})}{(1 - 2Z^{-1} + Z^{-2})(0.368Z^{-1})(1 + 0.713Z^{-1})}$$

$$= 5.44 \frac{(1 - 0.868Z^{-1} + 0.184Z^{-2})}{(1 - 0.287Z^{-1} - 0.713Z^{-2})}$$

These controllers were then used in the system of fig. 2.18 and the results are shown in tables 2.3 to 2.5.

1. <u>Dead-beat controller for ramp disturbances</u>

$$D(Z) = 5.44 \frac{(1 - 0.868Z^{-1} + 0.184Z^{-2})}{(1 - 0.287Z^{-1} - 0.713Z^{-2})}$$

$T = 1$ sec.

Max. overshoot to a step disturbance = 150%

Settling time for a ramp disturbance = 3 T

Settling time for a step disturbance = 13 T

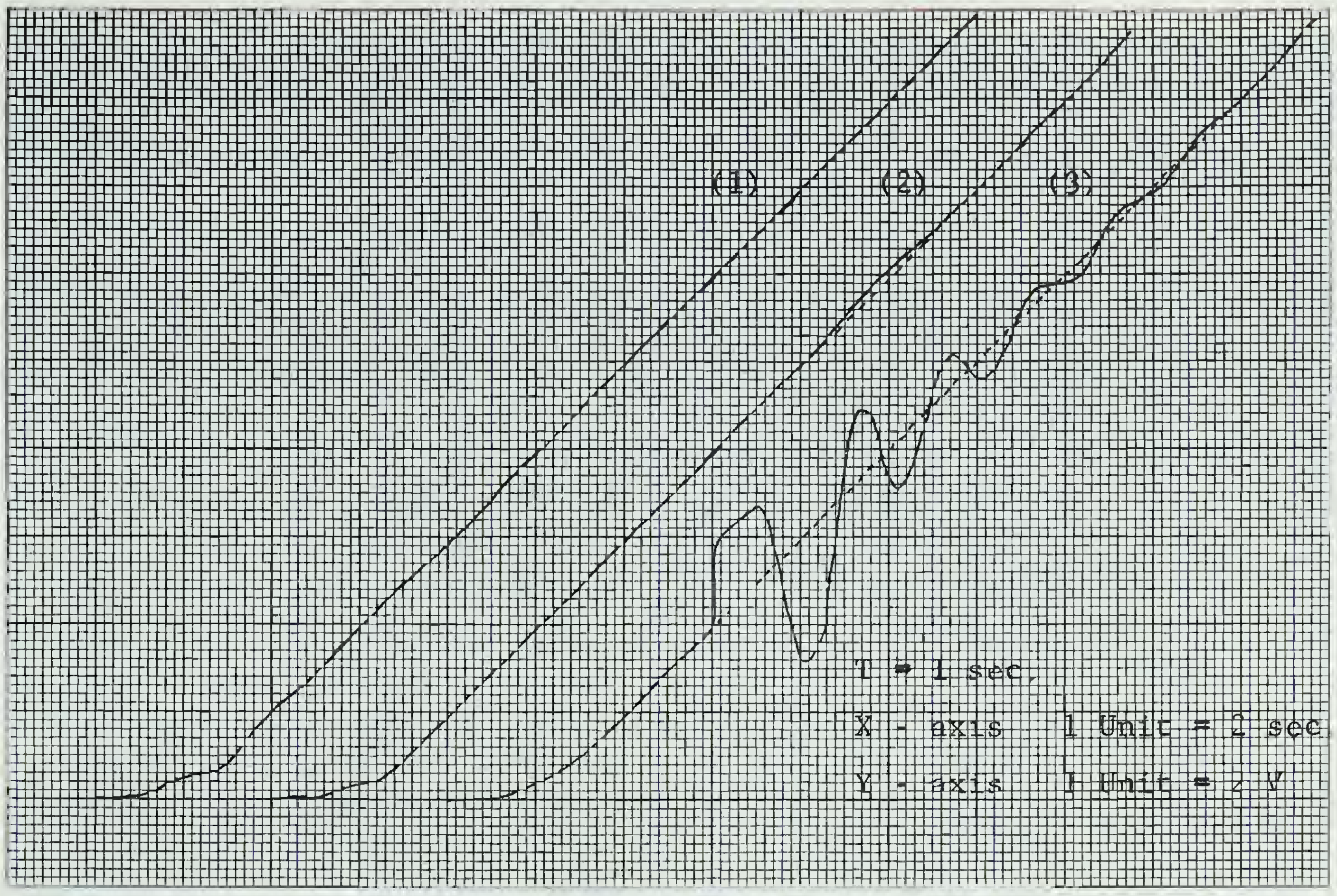


Fig. 2.21

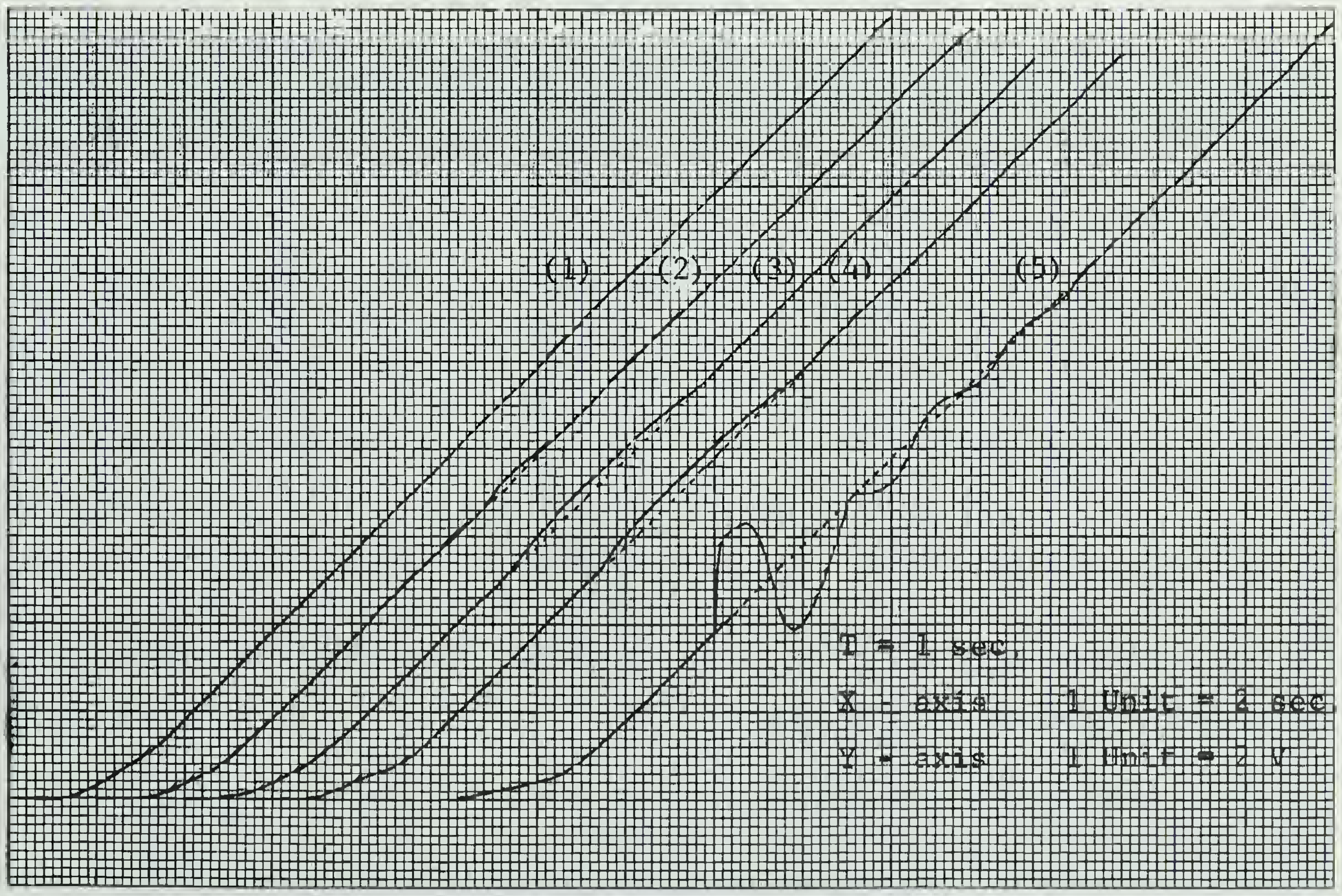


Fig. 2.22

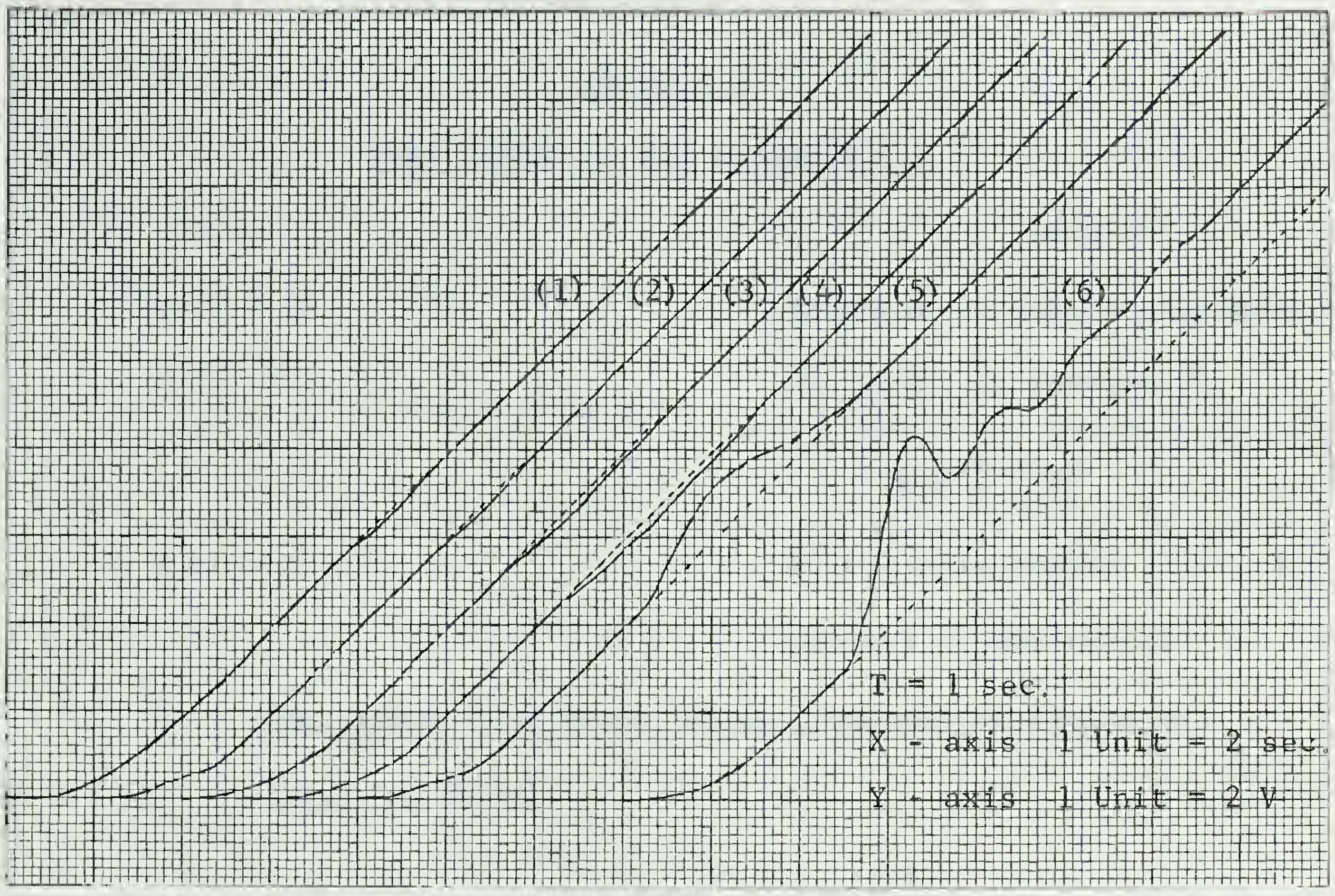


Fig. 2.23

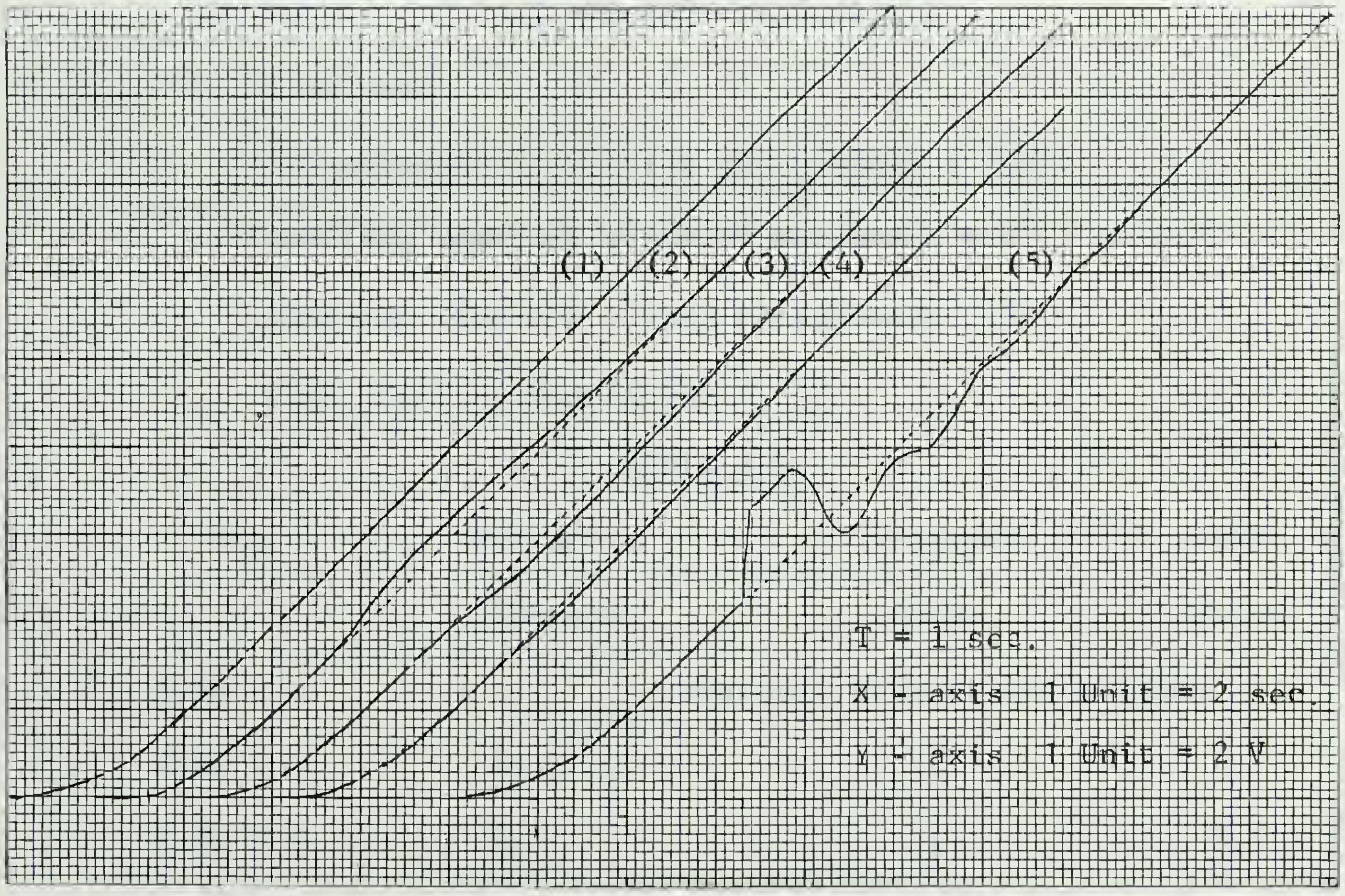


Fig. 2.24

TABLE 2.3

STEP INPUT TO THE SYSTEM = 2 V

Fig.	Remarks
2.21 - 1	Response without disturbance
2.21 - 2	the plant is changed to $\frac{1}{S(S + 1.5)}$
2.21 - 3	A step disturbance of 2 V is applied at the output

2. The closed loop poles of the compensated system at (0.27 + j 0.54)

$$D(Z) = 3.967 \frac{(1 - 0.931Z^{-1} + 0.207Z^{-2})}{(1 - 0.267Z^{-1} - 0.713Z^{-2})}$$

T = 1 sec.

Max overshoot to a step disturbance = 100%

Settling time for a step disturbance = 8T

Settling time for a ramp disturbance = 4T

Step input to the system = 2 V.

TABLE 2.4

Fig.	Parameter Changed	Original Value	New Value
2.22 - 1	Response without disturbance		
2.22 - 2	K_2	1	1.1
2.22 - 3	K_2	1	1.3
2.22 - 4	K_2	1	1.5

Fig.	Parameter Changed	Original Value	New Value
2.22 - 5	Step disturbance of 2 V at the output		
2.23 - 1 & 2	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+1.1)}$
2.23 - 3	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+1.3)}$
2.23 - 4	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+1.5)}$
2.23 - 5	A step disturbance of 2 V is applied at the point '1' in fig. 2.17.		
2.23 - 6	A step disturbance of 2 V is applied at the point 'm' in fig. 2.17.		

3. <u>The closed loop poles of the compensated system at $(0.5 \pm j\,0.15)$</u>

$$D(Z) = 2.72 \frac{(1 - 1.095Z^{-1} + 0.268Z^{-2})}{(1 - 0.287Z^{-1} - 0.713Z^{-2})}$$

T = 1 sec.

Max. overshoot to a step disturbance = 38%

Settling time for a step disturbance = 13 T

Settling time for a ramp disturbance = 8 T

Step input to the system = 2 V

TABLE 2.5

Fig.	Parameter Changed	Original Value	New Value
2.24 - 1	Response without disturbance		

Fig	Parameter Changed	Original Value	New Value
2.24 - 2	K_2	1	1.5
2.24 - 3	Time constant	$\frac{1}{S(S + 1)}$	$\frac{1}{S(S + 1.5)}$
2.24 - 4	Time constant	$\frac{1}{S(S + 1)}$	$\frac{1}{S(S + 1.2)}$
2.24 - 5	A step disturbance of 2 V is applied at the output.		

Here again the change of parameters was affected instantaneously by switches. In actual practice, they are likely to occur gradually and thus the output will be less affected.

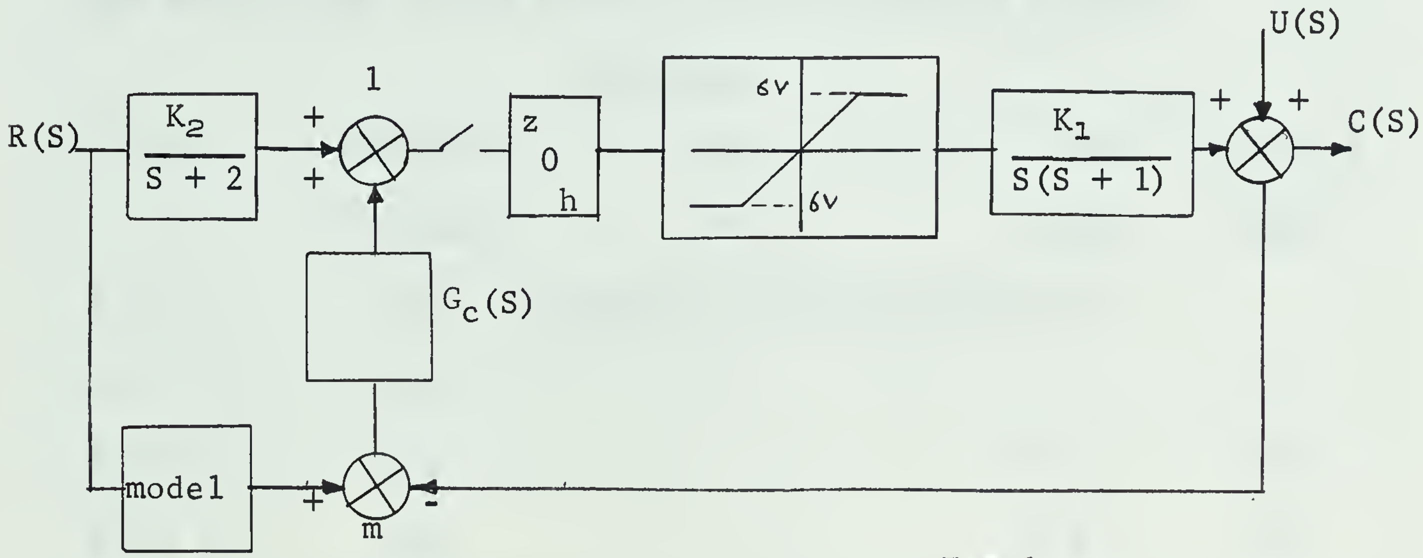
A ramp input was then applied to the system. It gave a good parabolic response. The error in the branch of $G_c(S)$ diminished to zero. The controller effectively removed step disturbances at the output. Any parameter change was equivalent to a parabolic disturbance and since the controller had been designed for ramp disturbances, it gave a constant error to acceleration disturbances. If, however, ramp inputs are generally expected into the system, the controller could be designed for parabolic disturbances and then modified to give a good response to ramp and step disturbances.

CHAPTER III

This chapter deals with non-linear sampled data systems.

3.1 SATURATION TYPE NON-LINEARITY BEFORE THE PLANT

The transfer function of the plant chosen in this case was $\frac{K_1}{S(S+1)}$. The conditional feedback arrangement is shown in fig. 3.1.



T = 1 sec.

Fig. 3.1

Step input into the system was considered. In this case the model was simulated exactly similar to the actual system i.e. it included a sampler, a zero order hold and the non-linearity. A continuous model could not be found, because there is no transform for the non-linearity.

The dead-beat digital controller was designed by the rep-op method for a ramp disturbance of 2 V/sec. and the non-linearity set at 6 V, the dead-beat controller is

$$D(Z) = 4.087 \frac{1 - 0.697Z^{-1} - 0.178Z^{-2} + 0.1046Z^{-3}}{1 - 0.05Z^{-1} - 0.675Z^{-2} - 0.275Z^{-3}}$$

This system was then simulated on the analog computer to study the effect of changes of parameters on the output. The system was

observed to behave very well. The various disturbances studied are shown in table 3.1.

Step input to the system = 2 V

Settling time refers to the time required for the output to return to its normal value after the application of the disturbance.

The small blips on the curves show the sampling instants.

TABLE 3.1

Fig.	Parameter Changed	Original Value	New Value	Settling Time
3.2 - 1	Response without disturbance			
3.2 - 2	K_2	1	1.1	2T
3.2 - 3	K_2	1	1.5	5T
3.2 - 4	K_2	1	2.0	7T
3.2 - 5	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+1.1)}$	4T
3.2 - 6	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+1.5)}$	5T
3.2 - 7	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+2)}$	7T
3.2 - 8	A step disturbance of 5 V is applied at the output			14T
	STEP INPUT TO THE SYSTEM = 5 V			
3.3 - 1	Response without disturbance			
3.3 - 2	K_2	1	2.0	8T
3.3 - 3	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+2)}$	13T
3.3 - 4	A step disturbance of 5 V is applied at the output			11T

Fig.	Parameter Changed	Original Value	New Value	Settling Time
	STEP INPUT TO THE SYSTEM = 8 V			
3.4 - 1	Response without disturbance			
3.4 - 2	K_2	1	2.0	8T
3.4 - 3	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+2)}$	9T
3.4 - 4	A step disturbance of 5 V is applied at the output			12T
	STEP INPUT TO THE SYSTEM = 10 V			
3.5 - 1	Response without disturbance			
3.5 - 2	K_2	1	2.0	9T
3.5 - 3	K_2	1	0.5	6T
3.5 - 4	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+1.15)}$	4T
3.5 - 5	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+1.2)}$	4T
3.5 - 6	A step disturbance of 5 V is applied at the output			19T

When the plant was changed to $\frac{1}{S(S+1.25)}$, the disturbance could not be removed. The error kept on increasing in the branch of the controller.

It may be appropriate now to make the following comments about the controller.

1. The controller was designed for ramp disturbances only. It was discussed in chapter two that a change in parameter is equivalent to the sum of exponential, step and ramp disturbances at

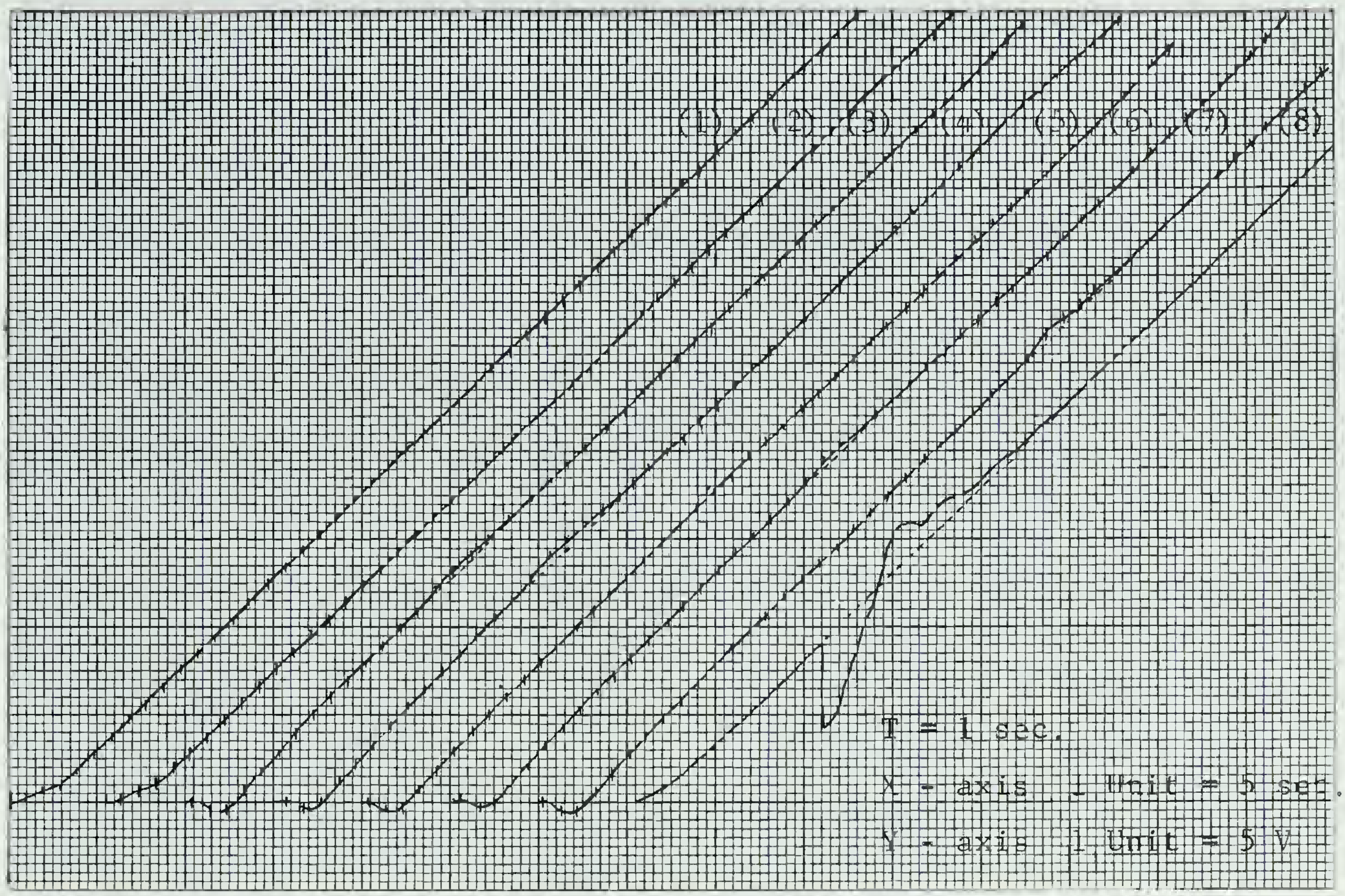

(1)
(2)
(3)
(4)
(5)
(6)
(7)
(8)
T = 1 sec.
X - axis 1 Unit = 5 sec.
Y - axis 1 Unit = 5 V

Fig. 3.2

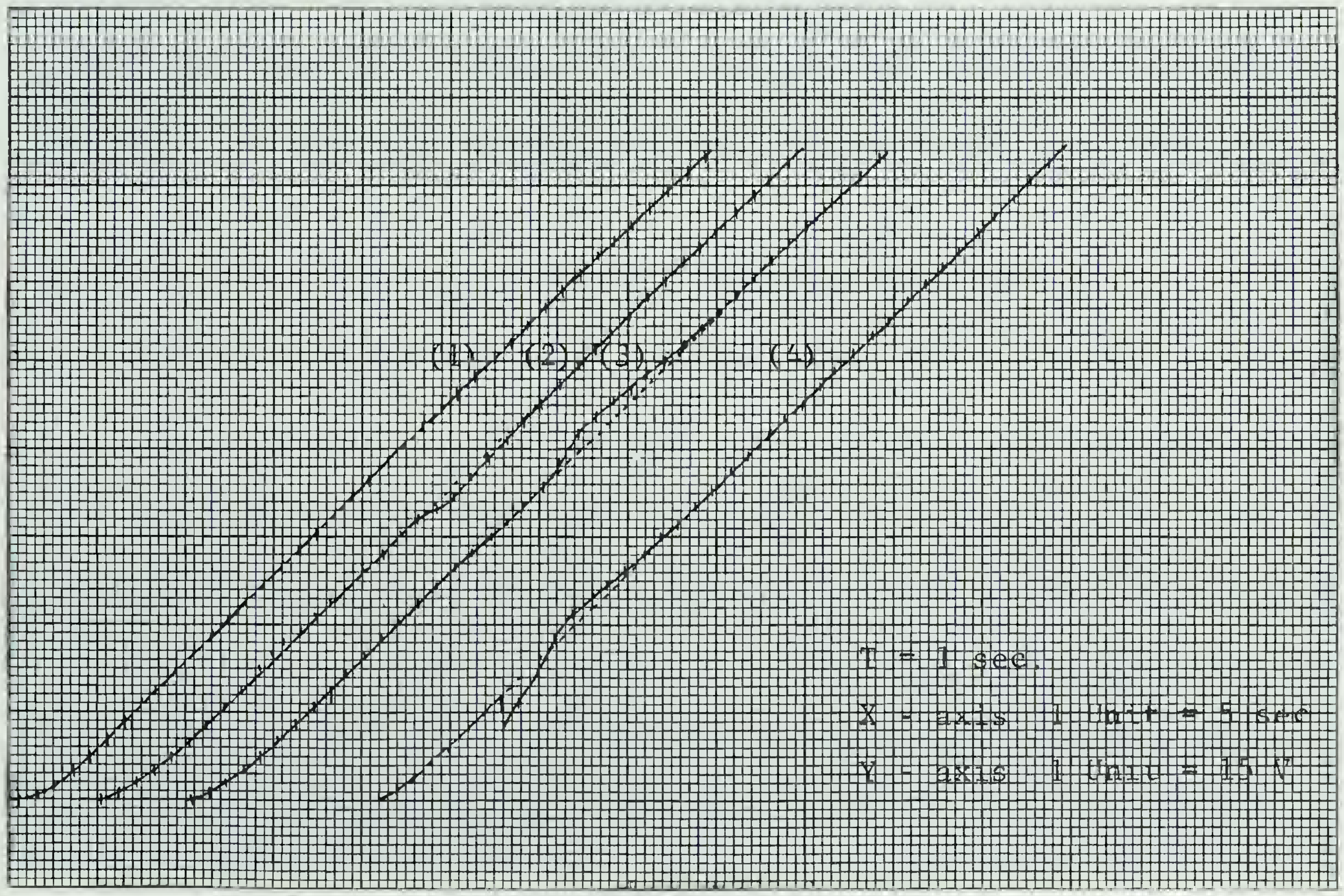

(1)
(2)
(3)
(4)
T = 1 sec.
X - axis 1 Unit = 5 sec
Y - axis 1 Unit = 15 V

Fig. 3.3

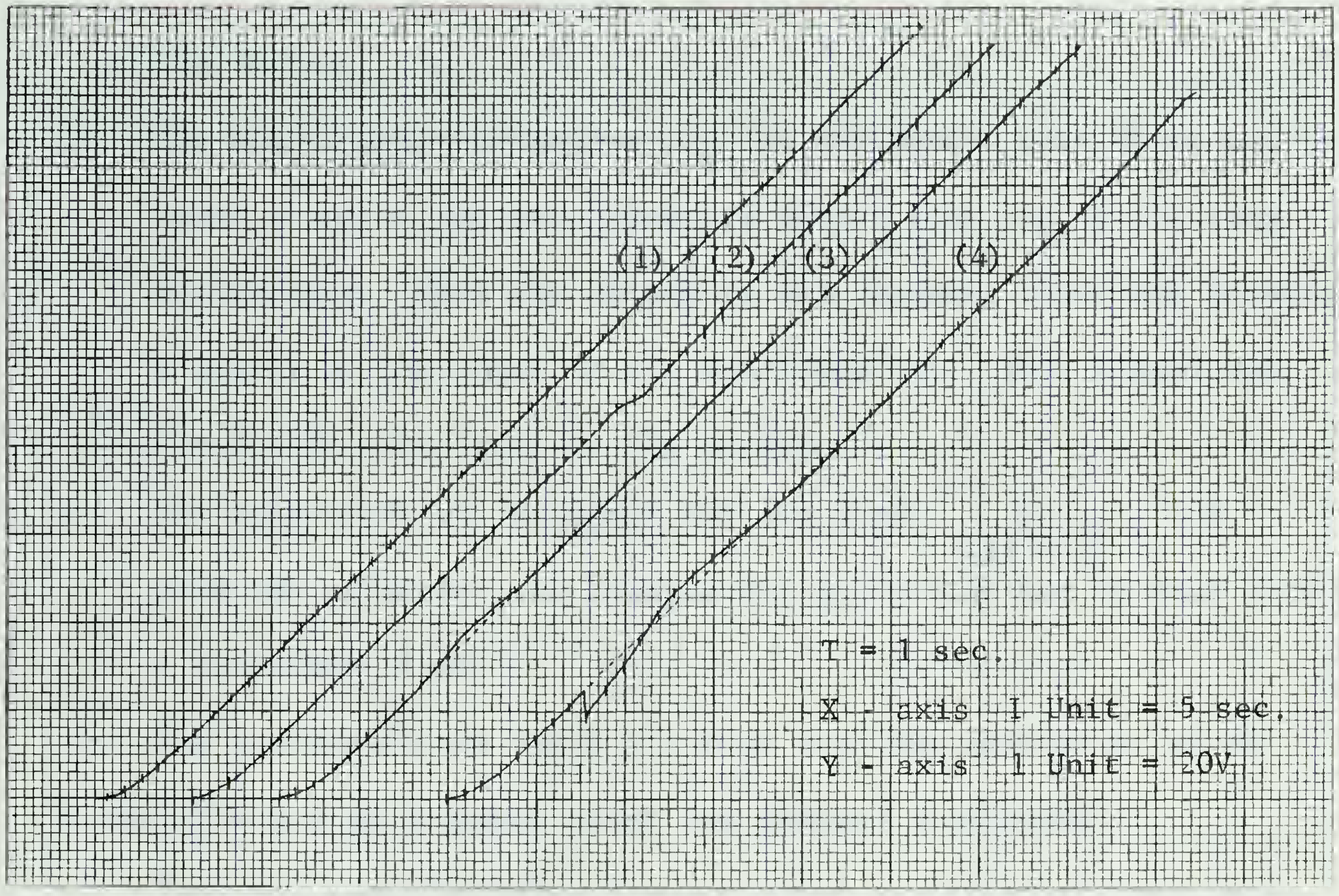

(1)
(2)
(3)
(4)
T = 1 sec.
X - axis 1 Unit = 5 sec.
Y - axis 1 Unit = 20V

Fig. 3.4

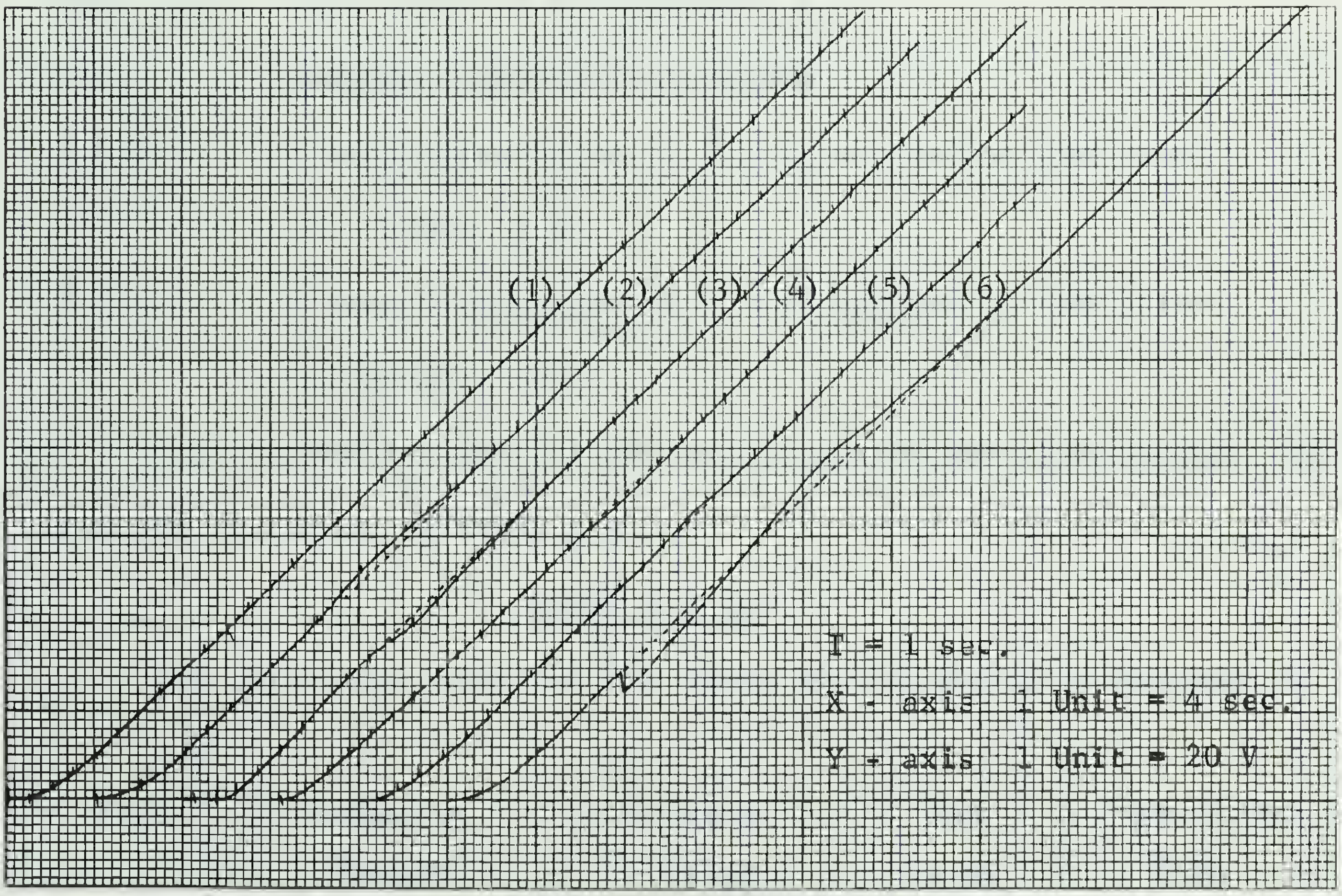

(1)
(2)
(3)
(4)
(5)
(6)
T = 1 sec.
X - axis 1 Unit = 4 sec.
Y - axis 1 Unit = 20 V

Fig. 3.5

the output.

2. The controller was designed on the assumption that there was no other input to the non-linearity except that due to the ramp disturbance. When the parameter was changed, there was already a signal at the input to the non-linearity. This signal was due to the step input to the system.

3. The controller was designed for the plant $\frac{1}{S(S+1)}$, but when the parameter was changed the controller had to deal with a different plant.

The results in the table 3.1 show that the controller gives a satisfactory performance under the changed conditions.

Next the step disturbance was applied at different locations

Fig. 3.6 : Step input to the system = 2 V

3.6 - 1 : Response without disturbance

3.6 - 2 : A step disturbance of 5 V is applied at the point '1' in fig. 3.1

3.6 - 3 : A step disturbance of 5 V is applied at the point 'm' in fig. 3.1.

The saturation limit of the non-linearity was then varied. The results are shown in table 3.2.

TABLE 3.2 (a)

The saturation limit of the plant non-linearity is changed to 5 V.

Fig.	Parameter Changed	Original Value	New Value	Settling Time
3.7 - 1	Response without disturbance			

Fig.	Parameter Changed	Original Value	New Value	Settling Time
3.7 - 2	A step disturbance of 5 V is applied at the output			13T
3.7 - 3	K_2	1	2.0	8T
3.7 - 4	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+2)}$	8T
3.7 - 5	K_2	1	0.5	6T
3.7 - 6	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+0.5)}$	6T

TABLE 3.2 (b)

The saturation limit of the plant non-linearity is changed to 7 V.

THE STEP INPUT TO THE SYSTEM = 2 V

Fig.	Parameter Changed	Original Value	New Value	Settling Time
3.8 - 1	Response without disturbance			
3.8 - 2	A step disturbance of 5 V is applied at the output			18T
3.8 - 3	K_2	1	0.5	7T
3.8 - 4	K_2	1	2.0	8T
3.8 - 5	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+2)}$	9T
3.8 - 6	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+0.5)}$	———

When the plant was changed to $\frac{1}{S(S+0.5)}$ the oscillations observed at the output exhibited a slow decay.

Table 3.1 shows that the settling time for the disturbance

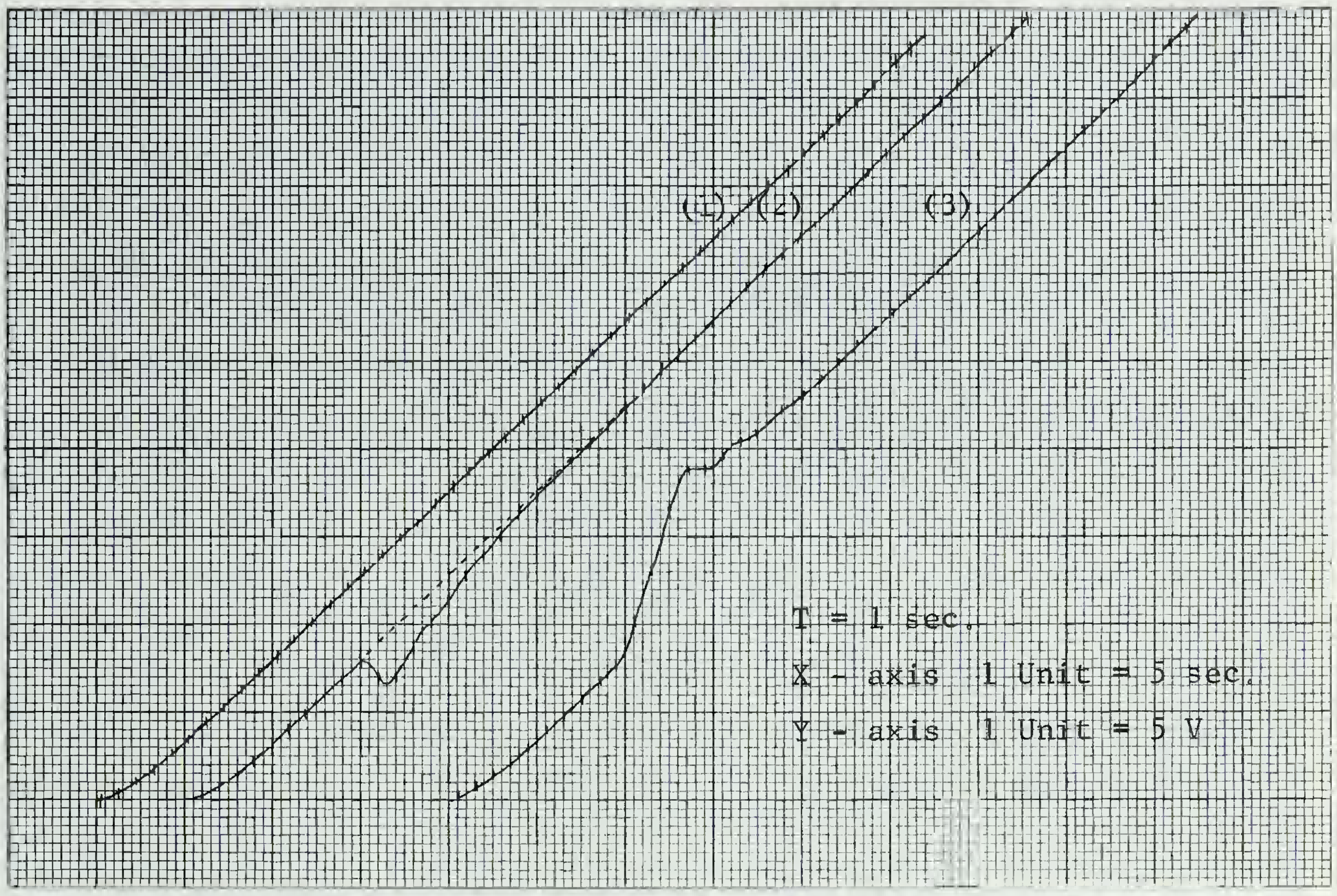

(1)
(2)
(3)
T = 1 sec.
X - axis 1 Unit = 5 sec.
Y - axis 1 Unit = 5 V

Fig. 3.6

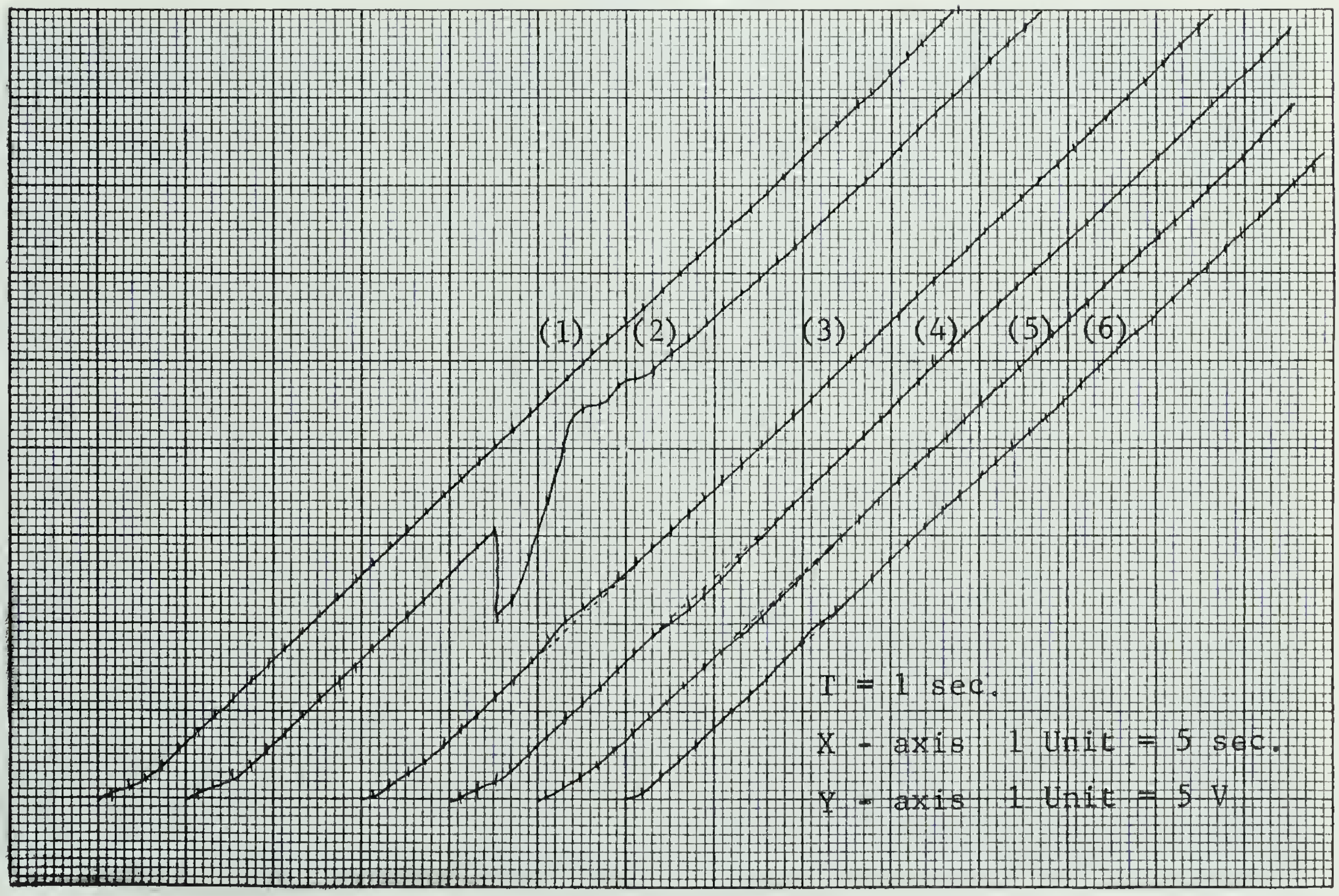

(1)
(2)
(3)
(4)
(5)
(6)
T = 1 sec.
X - axis 1 Unit = 5 sec.
Y - axis 1 Unit = 5 V

Fig. 3.7

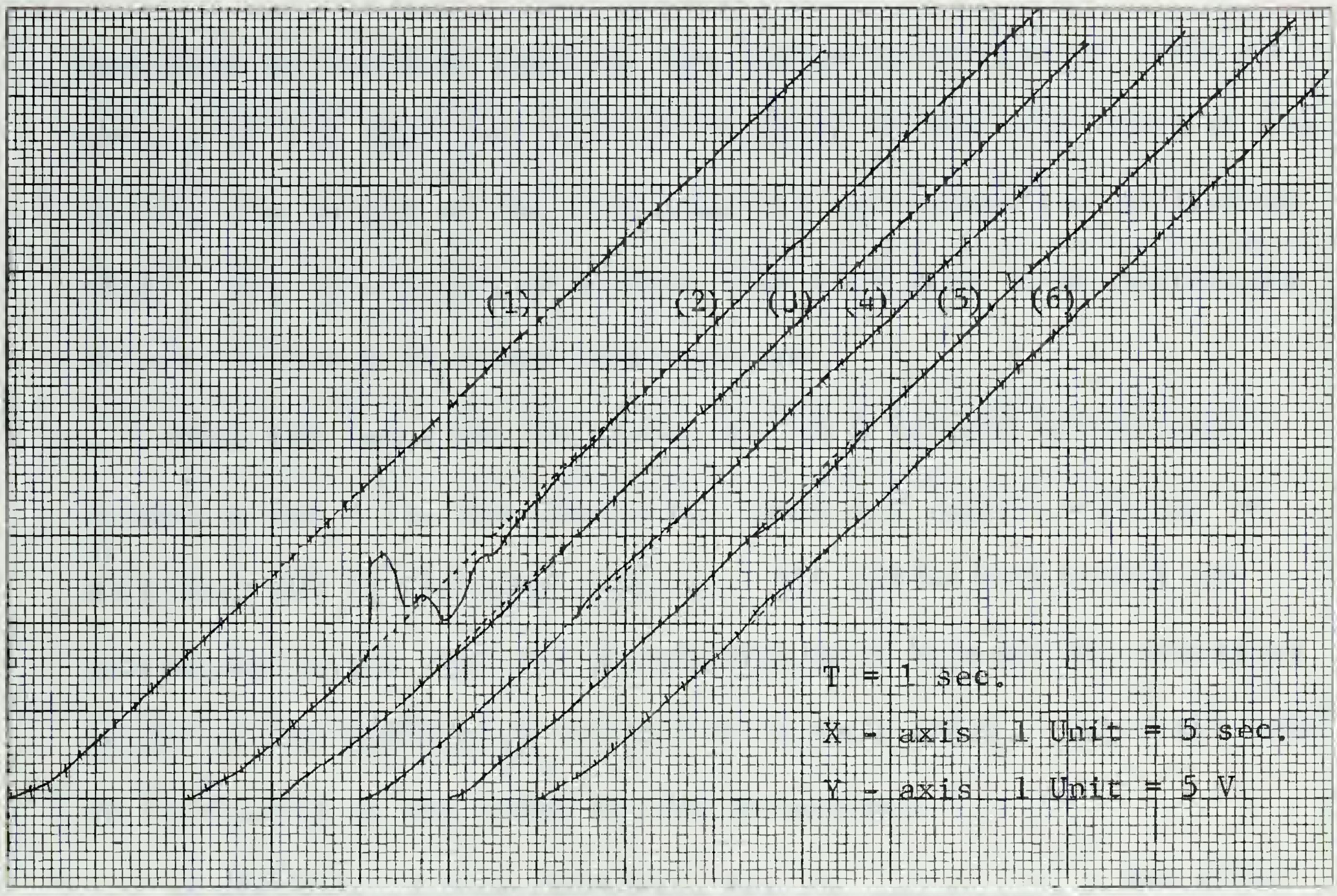


Fig. 3.8

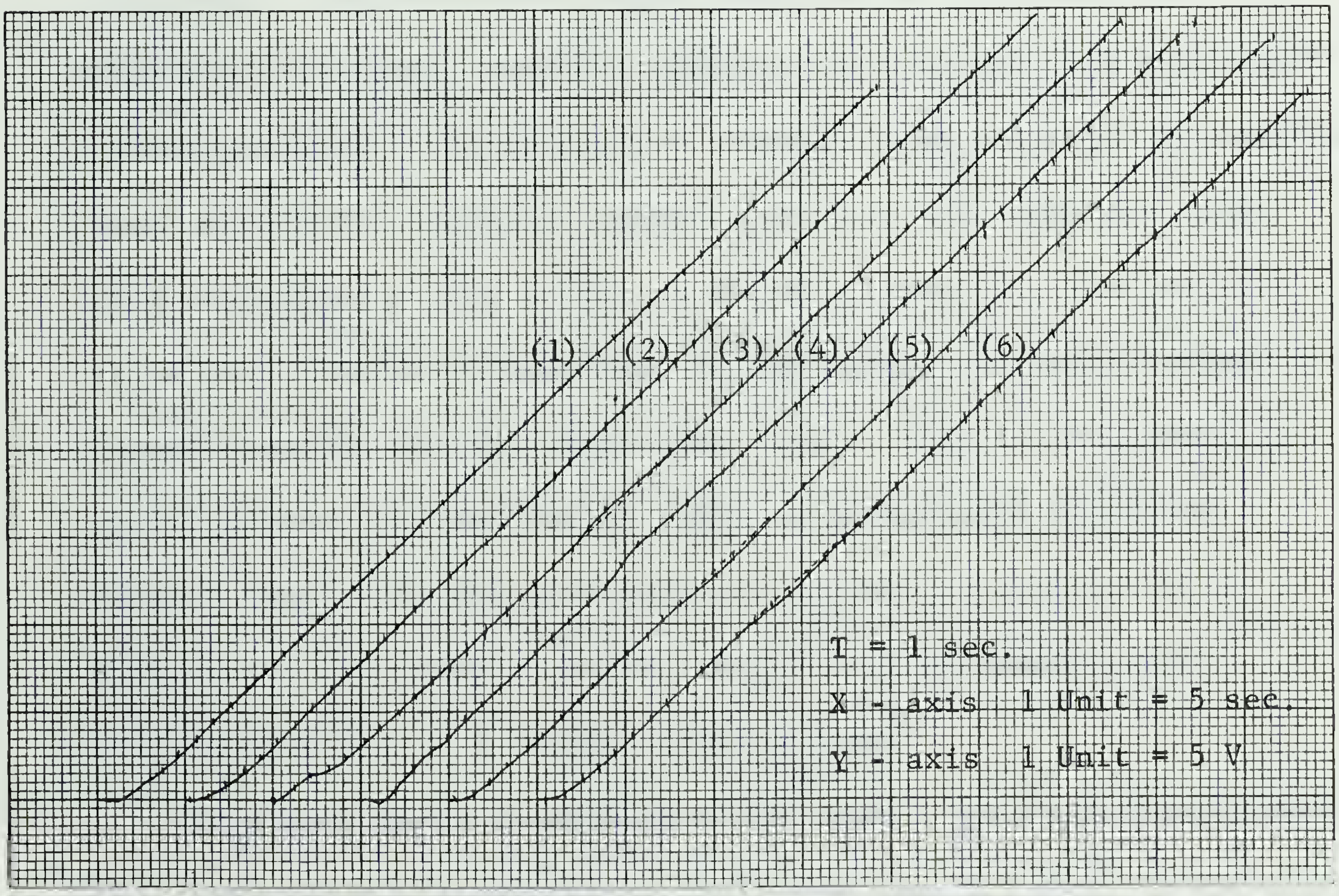


Fig. 3.10

increases as the step input to the system is increased. This is because of the fact that with an increase in the step input to the system, the output of the non-linearity approaches the saturation limit. The non-linearity is then unable to give large corrective outputs required to remove disturbances.

When the input to the system was 10 V (signal at the non-linearity was 5 V), the controller could not remove a change of more than 20% in the time constant. The corrective signal required was larger than the margin available between the operating point and the saturation limit (1 V in this case). The error kept on increasing at the input to the controller.

A decrease in time constant, say to $\frac{1}{S(S + 0.5)}$, was still tolerated. The corrective signal required in this case was negative. This was, however, achieved at the cost of a more oscillatory response. The controller produces both positive and negative steps during the transient response, whereas the negative steps were reproduced at the output of the non-linearity, the positive steps were limited in magnitude to the margin available between the operating point and the saturation limit. Thus if the system is operating near the saturation limit, a disturbance requiring a larger output from the non-linearity may not be removed while a disturbance requiring a smaller output will be removed with oscillations and in a longer settling time.

If a ramp input is applied to the above system, the non-linearity will soon reach its saturation limit and then the above discussion applies. The system is thus not very good for ramp inputs or for

those step inputs which make the non-linearity to operate near its saturation limit. This will be true for a conventional feedback system also and is thus not a drawback of the conditional feedback system alone.

3.2 SATURATION TYPE NON-LINEARITY IN BETWEEN TWO TRANSFER FUNCTIONS

The saturation type non-linearity was then placed as shown in fig. 3.9.

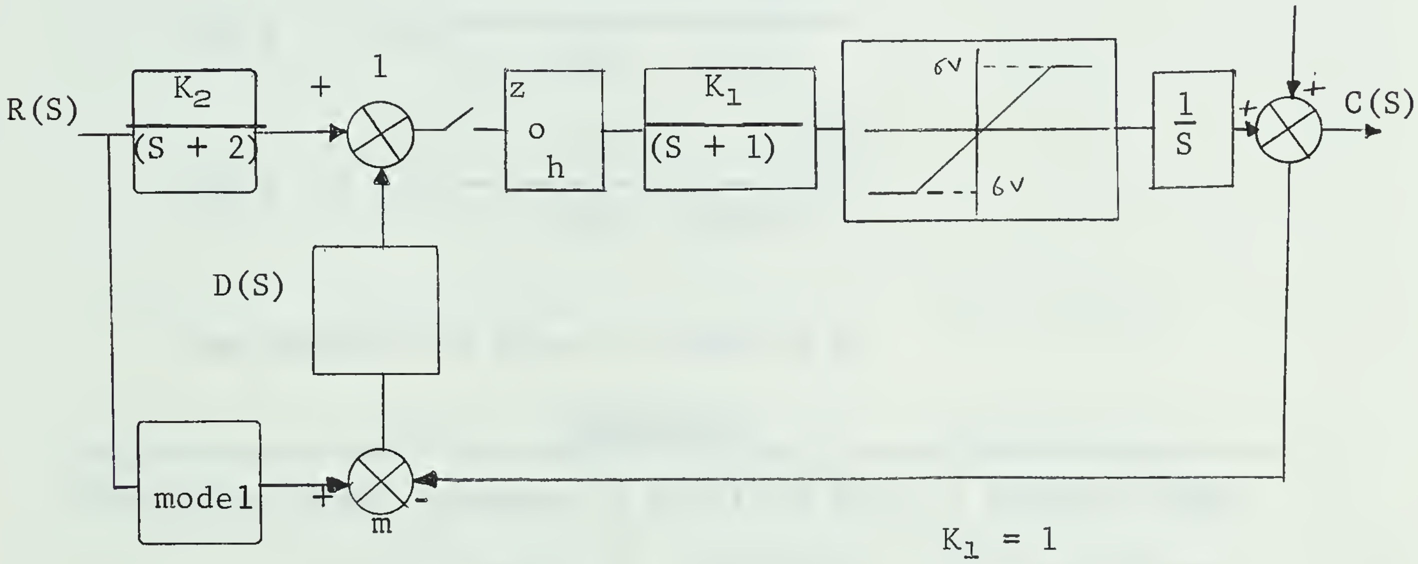


Fig. 3.9

$K_1 = 1$
$K_2 = 1$
T = 1 sec.

The dead-beat controller was designed by the rep-op method. In a case like this, the design of a dead-beat controller by the state variable approach will be quite involved but it was quite simple by the rep-op method.

The dead-beat controller for a ramp disturbance of 2 V and the saturation limit of the non-linearity set at 6 V was found to be

$$D(Z)_1 = 3.922 \frac{1 - 1.013\bar{Z}^1 + 0.252\bar{Z}^2}{1 - 0.5\,\bar{Z}^1 - 0.5\bar{Z}^2}$$

T = 1 sec.

This controller was then modified by trial to give a good re-

sponse to a step input also. The coefficients of $\bar{Z}^1$ and $\bar{Z}^2$ in the numerator were varied. The various controllers studied were as follows

$$D(Z)_2 = 3.922 \frac{1 - 1.1\bar{Z}^1 + 0.260\bar{Z}^2}{1 - 0.5\bar{Z}^1 - 0.5\bar{Z}^2}$$

$$D(Z)_3 = 3.922 \frac{1 - 1.15\bar{Z}^1 + 0.270\bar{Z}^2}{1 - 0.5\bar{Z}^1 - 0.5\bar{Z}^2}$$

$$D(Z)_4 = 3.922 \frac{1 - 1.2\bar{Z}^1 + 0.280\bar{Z}^2}{1 - 0.5\bar{Z}^1 - 0.5\bar{Z}^2}$$

$$D(Z)_5 = 3.922 \frac{1 - 1.125\bar{Z}^1 + 0.265\bar{Z}^2}{1 - 0.5\bar{Z}^1 - 0.5\bar{Z}^2}$$

The results are shown in table 3.3.

TABLE 3.3

Controller	Max. Overshoot to a step input	Settling Time for a step input	Settling Time for a ramp input
$D(Z)_1$	80%	10T	3T
$D(Z)_2$	60%	11T	9T
$D(Z)_3$	42%	14T	12T
$D(Z)_4$	30%	28T	20T
$D(Z)_5$	50%	13T	10T

$D(Z)_5$ was finally chosen.

This system was then simulated on the analog computer to study the effect of change of parameters on the output. The results are shown in table 3.4

TABLE 3.4

Fig.	Parameter Changed	Original Value	New Value	Settling Time
	STEP INPUT TO THE SYSTEM = 2 V			
3.10 - 1	Response without disturbance			
3.10 - 2	K_2	1	1.1	-
3.10 - 3	K_2	1	1.5	6T
3.10 - 4	K_2	1	2.0	9T
3.10 - 5	K_2	1	0.7	6T
3.10 - 6	K_2	1	0.5	7T
3.11 - 1	Response without disturbance			
3.11 - 2	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+1.1)}$	-
3.11 - 3	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+1.5)}$	8T
3.11 - 4	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+2)}$	8T
3.11 - 5	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+0.7)}$	7T
3.11 - 6	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+0.5)}$	12T
	STEP INPUT TO THE SYSTEM = 10 V			
3.12 - 1	Response without disturbance			
3.12 - 2	K_2	1	1,1	-
3.12 - 3	K_2	1	2.0	11T
3.12 - 4	K_2	1	0.5	6T
3.12 - 5	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+1.1)}$	3T

Fig.	Parameter Changed	Original Value	New Value	Settling Time
3.12 - 6	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+2)}$	9T
3.12 - 7	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+0.5)}$	14T

<u>STEP INPUT TO THE SYSTEM = 16 V</u>

Fig.	Parameter Changed	Original Value	New Value	Settling Time
3.13 - 1	Response without disturbance			
3.13 - 2	K_2	1	1.2	7T
3.13 - 3	K_2	1	2.0	13T
3.13 - 4	K_2	1	0.5	8T
3.13 - 5	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+1.2)}$	5T
3.13 - 6	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+2)}$	9T
3.13 - 7	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+0.5)}$	11T

It was observed in this case that it was possible to apply step inputs which make the non-linearity operate in its saturation region, but only the parameter variations before the non-linearity will be tolerated. The output of the model is 6 V/sec. This is also the maximum value of the output required from the plant. Therefore as long as the plant non-linearity has a saturation limit of 6 V or slightly more, the plant will always be able to yield an output of 6 V/sec. In fact those parameter variations which tend to produce larger signals at the input to the non-linearity may not be able to affect the output at all. The output of the non-linearity will stay at 6 V. This is, of course, true for a perfect non-linearity. As will be seen later, if the saturation

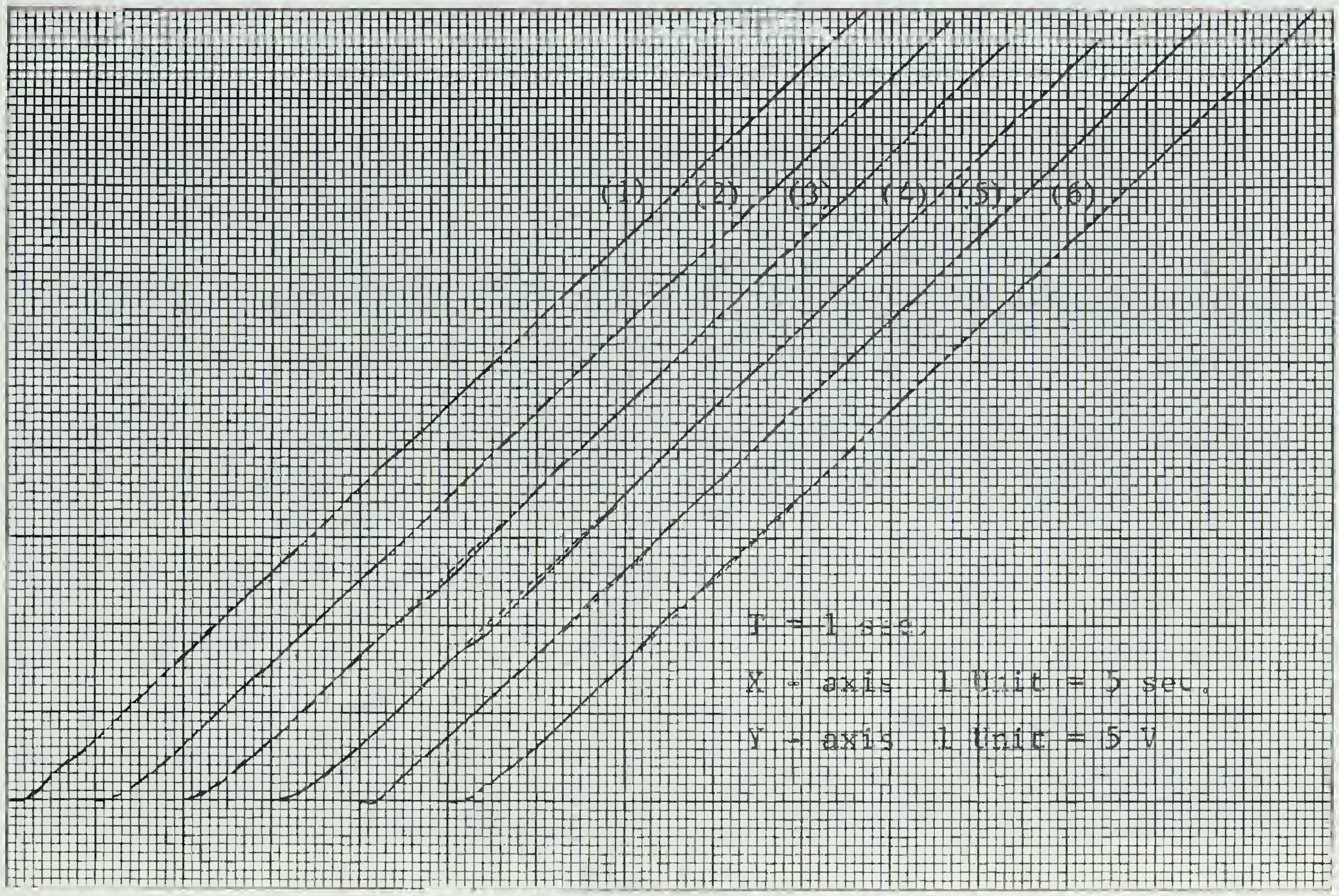


Fig. 3.11

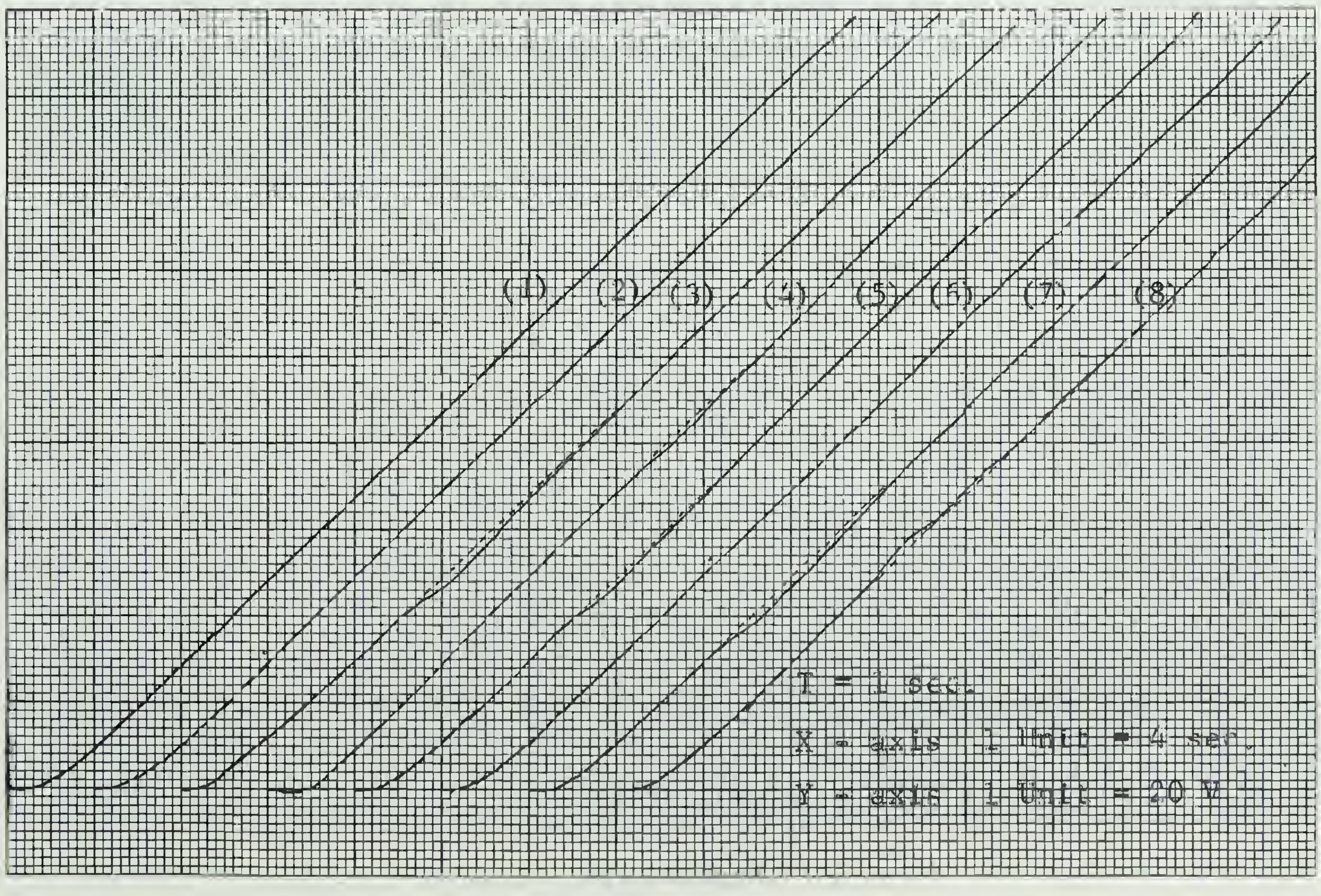


Fig. 3.12

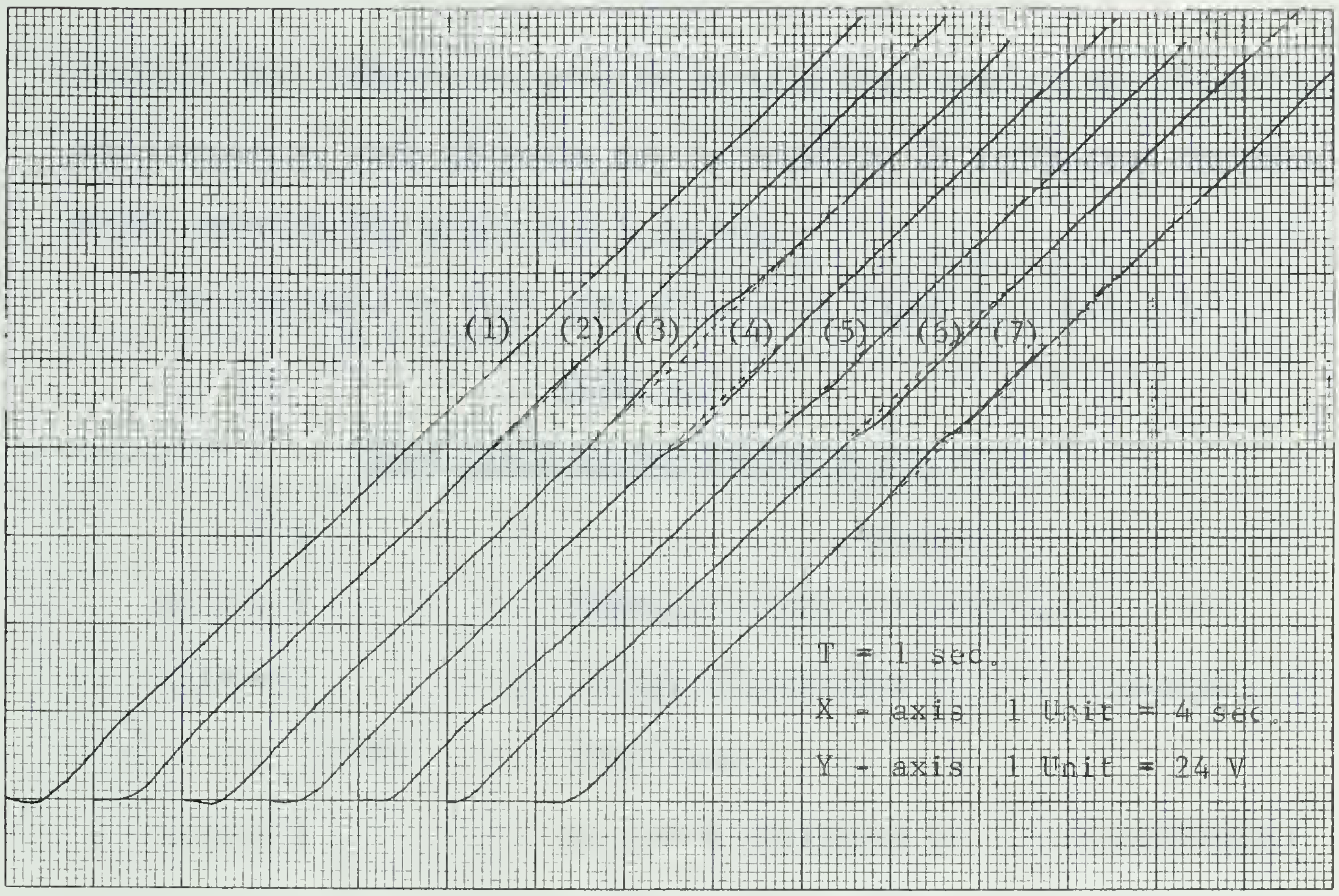


Fig. 3.13

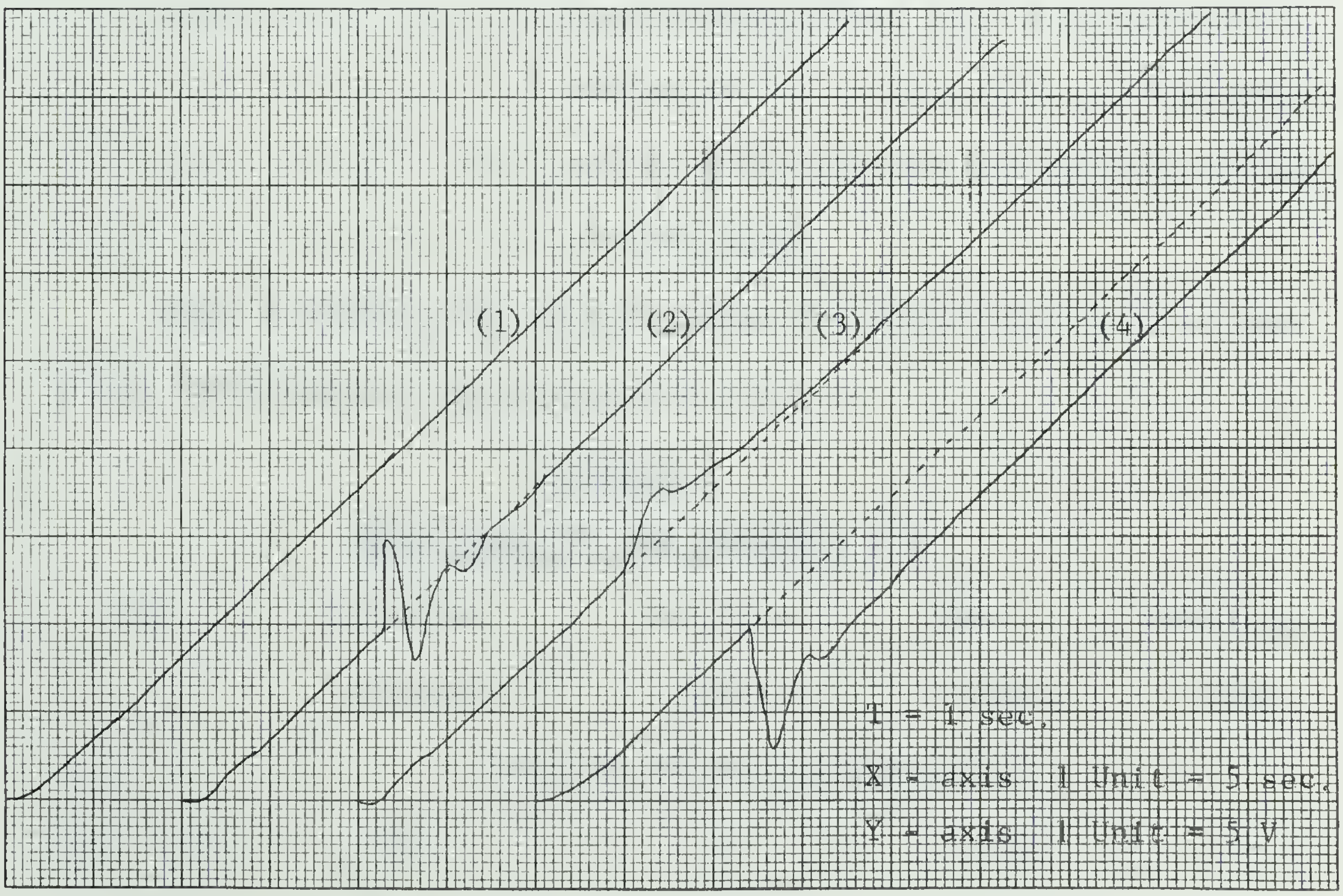


Fig. 3.14

limit of the plant non-linearity is less than that of the model non-linearity, then it would not be possible to apply step inputs which saturate the plant non-linearity.

The response to step disturbances is shown in table 3.5.

TABLE 3.5

STEP INPUT TO THE SYSTEM = 2 V

Fig.	Remarks
3.14 - 1	Response without disturbance
3.14 - 2	A step disturbance of 5 V is applied at the output
3.14 - 3	A step disturbance of 5 V is applied at the point 'l' in fig. 3.9
3.14 - 4	A step disturbance of 5 V is applied at the point 'm' in fig. 3.9. A disturbance at 'm' gives rise to an equal disturbance at the output

Table 3.6 shows the response when the saturation limit of the plant non-linearity was changed to 5 V.

TABLE 3.6

STEP INPUT TO THE SYSTEM = 2 V

Fig.	Parameter Changed	Original Value	New Value	Settling Time
3.15 - 1	Response without disturbance			
3.15 - 2	K_2	1	1.2	-
3.15 - 3	K_2	1	0.7	4T
3.15 - 4	Time constant	$\frac{1}{S(S + 1)}$	$\frac{1}{S(S + 2)}$	8T

Fig.	Parameter Changed	Original Value	New Value	Settling Time
3.15 - 5	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+1.2)}$	3T
3.15 - 6	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+0.7)}$	6T
3.15 - 7	A step disturbance of 5 V is applied at the output			11T

A step input of 8 V was then applied to the system and the above parameter variations were studied. The system behaved well in that case except that with large parameter changes, the settling time was larger.

A step input of 16 V will not work in this case, because the model will be giving a ramp output of 6 V/sec. and since the saturation limit in the plant non-linearity is 5 V, the plant output cannot exceed 5 V/sec. The error will therefore increase continuously in the branch of the controller.

The saturation limit of the non-linearity was then changed to 7 V. The responses to parameter variations are shown in table 3.7.

TABLE 3.7

STEP INPUT TO THE SYSTEM = 2 V

Fig.	Parameter Changed	Original Value	New Value	Settling Time
3.16 - 1	Response without disturbance			
3.16 - 2	K_2	1	1.2	-
3.16 - 3	K_2	1	1.5	7T
3.16 - 4	K_2	1	0.7	5T

Fig.	Parameter Changed	Original Value	New Value	Settling Time
3.16 - 5	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+1.2)}$	4T
3.16 - 6	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+0.7)}$	13T
3.16 - 7	A step disturbance of 5 V is applied at the output			10T
	STEP INPUT TO THE SYSTEM = 20 V			
3.17 - 1	Response without disturbance			
3.17 - 2	K_2	1	1.2	6T
3.17 - 3	K_2	1	2.0	16T
3.17 - 4	K_2	1	0.5	8T
3.17 - 5	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+1.2)}$	3T
3.17 - 6	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+2)}$	8T
3.17 - 7	Time constant	$\frac{1}{S(S+1)}$	$\frac{1}{S(S+0.5)}$	14T
3.17 - 8	A step disturbance of 20 V is applied at the output			-

In this case when the step input is applied to the system, the output of the plant tends to approach 7 V/sec. and that of the model to 6 V/sec. The controller starts acting and brings the output of the plant to 6 V/sec. This means that the plant is still operating in the linear range of the non-linearity and there is some margin available for additional signals to correct the disturbances. The settling time will, however, be larger than for the case when a smaller step input is applied to the system.

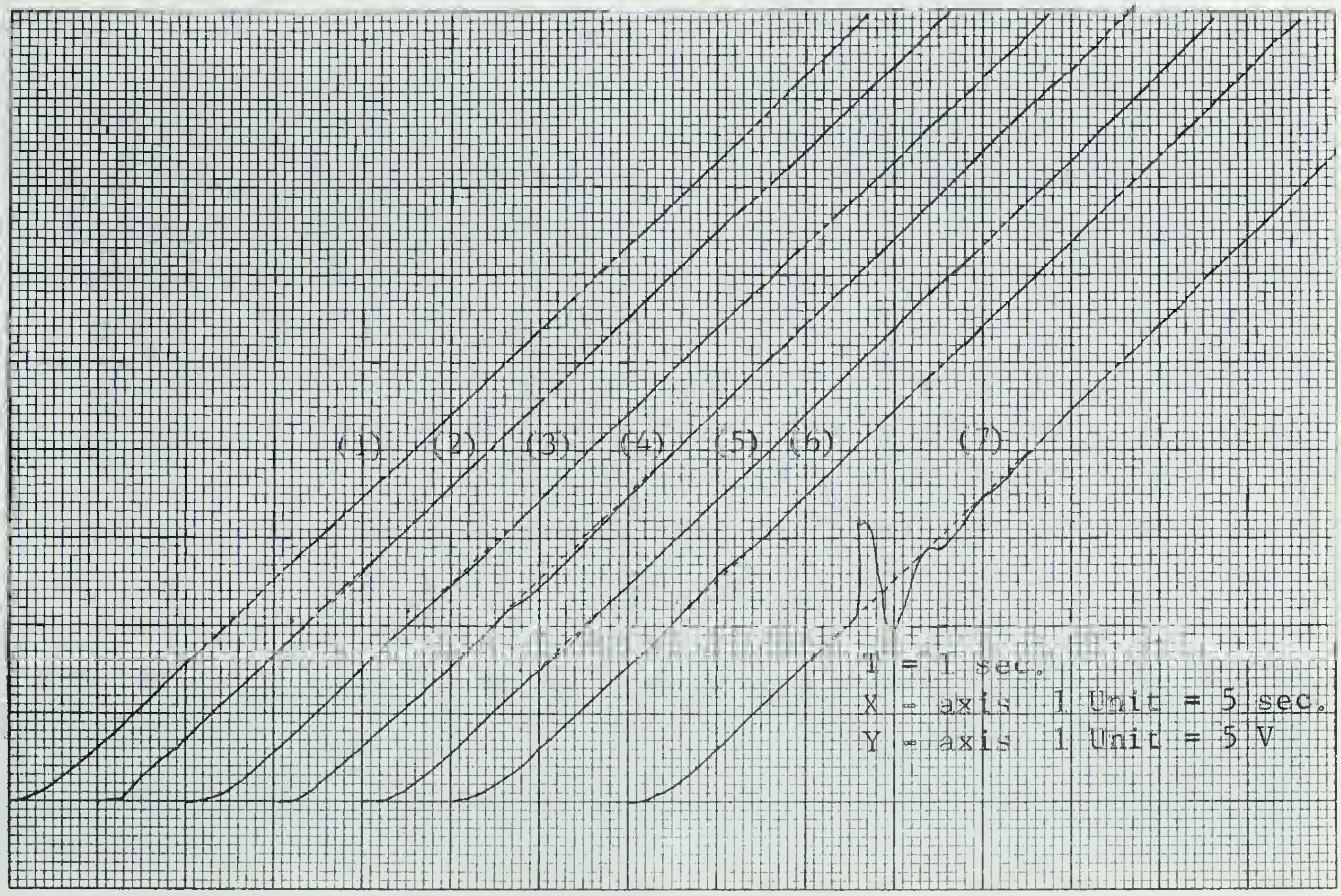


Fig. 3.15

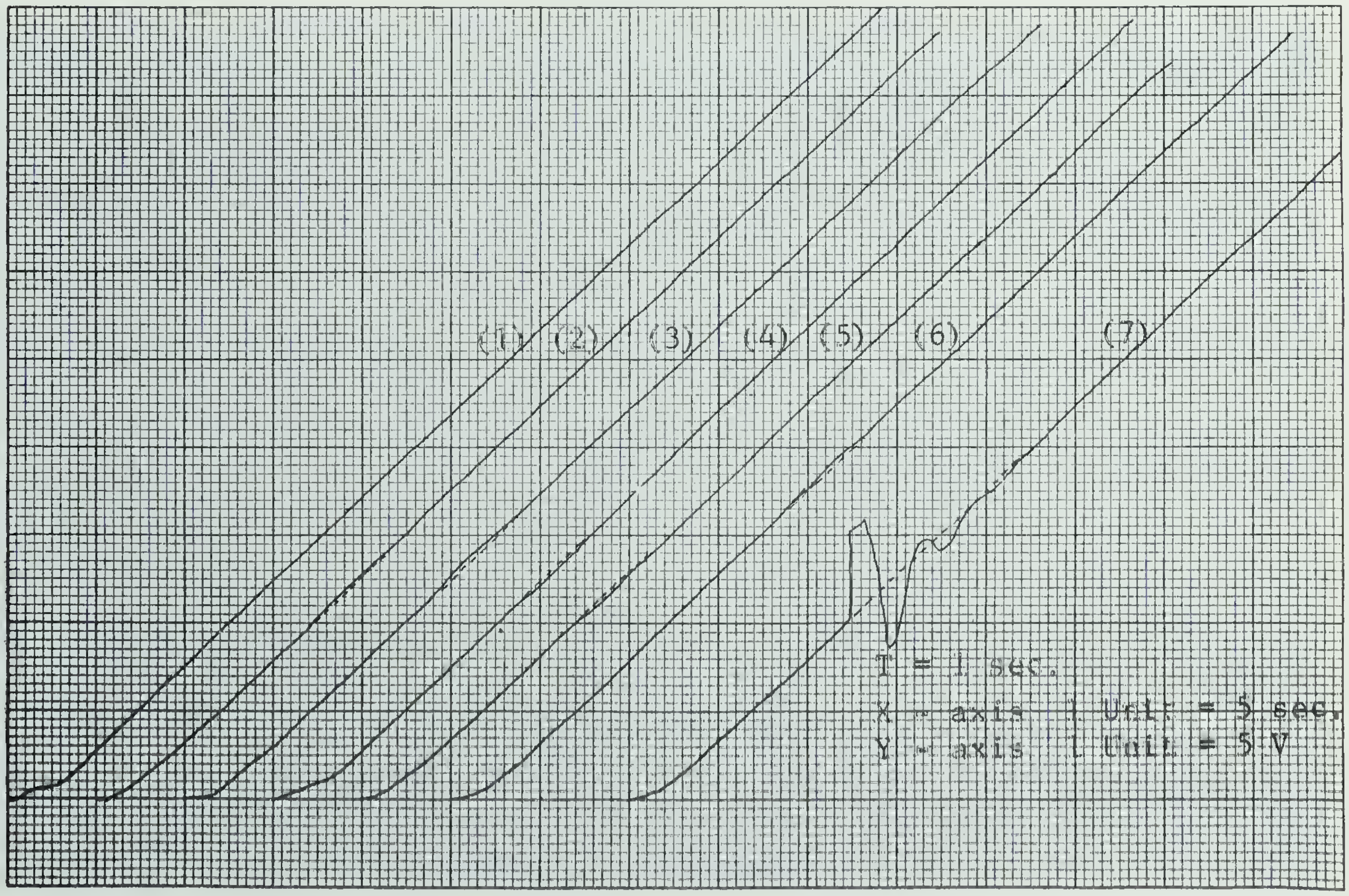


Fig. 3.16

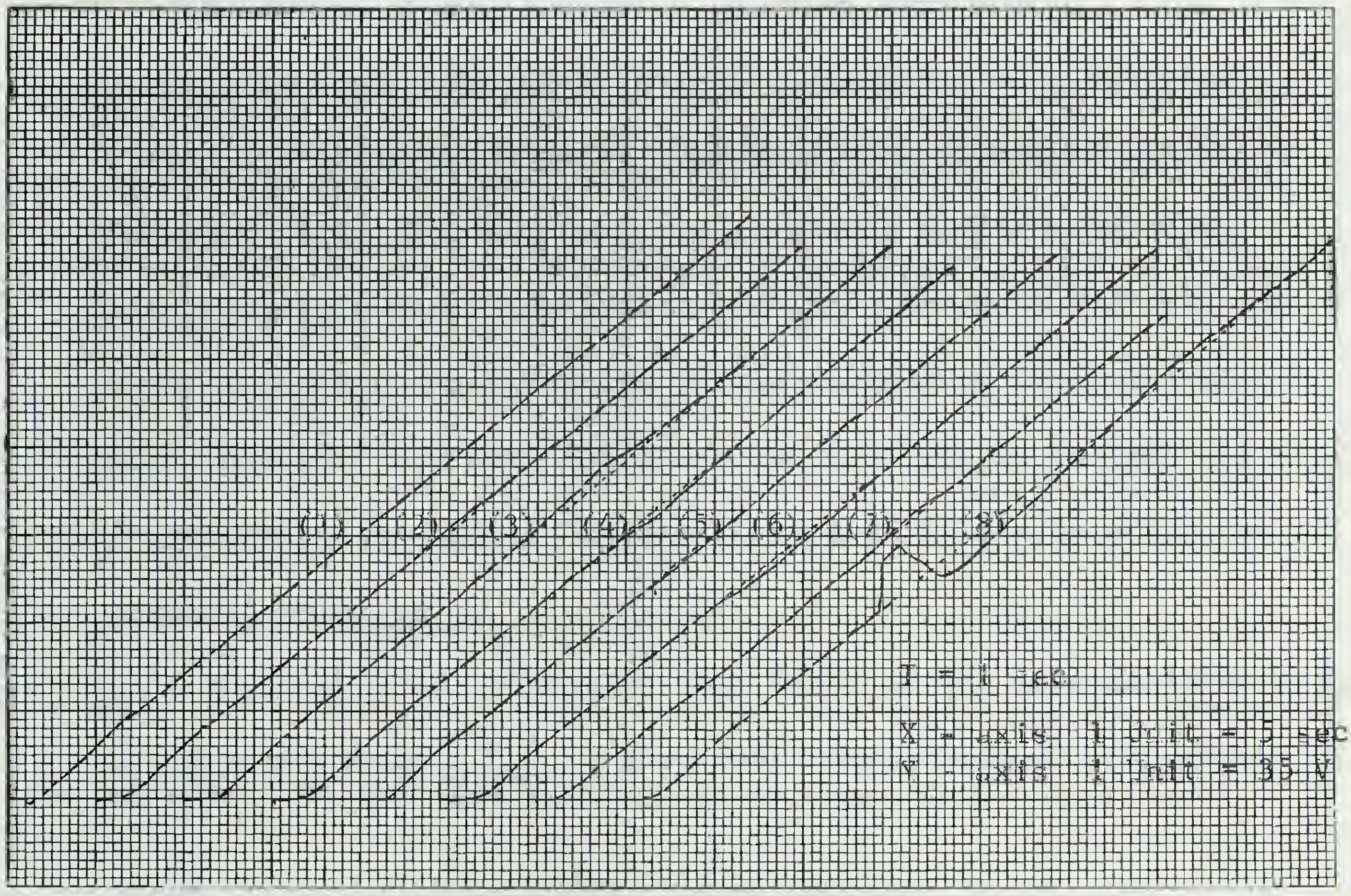


Fig. 3.17

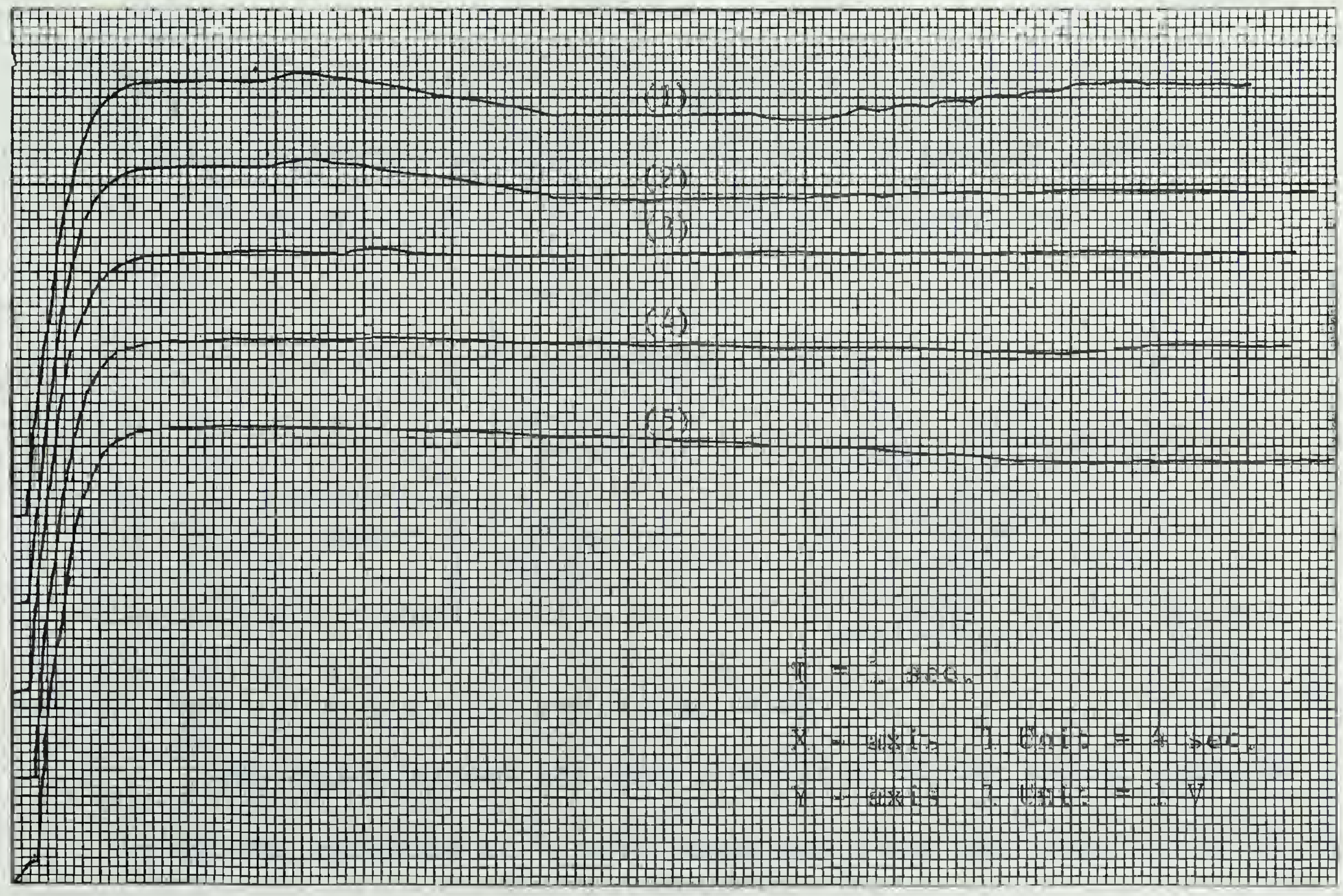


Fig. 3.17

3.3 HYSTERESIS TYPE NON-LINEARITY

This type of non-linearity is often encountered in the sensing devices. The hysteresis non-linearity was, therefore, placed in the feedback path. The model was simulated similar to the plant i.e. it contained a sampler, and a zero order hold, but no non-linearity was included in it. It was found that the model without a non-linearity worked as good as the model with a non-linearity. It was, therefore, decided not to include the non-linearity in the model. This will be true in practice because of economic reasons.

The dead-beat controller used here was the same as in article 2.1 for the plant $\frac{K_1}{S + 1}$. The conditional feedback arrangement is shown in fig. 3.19.

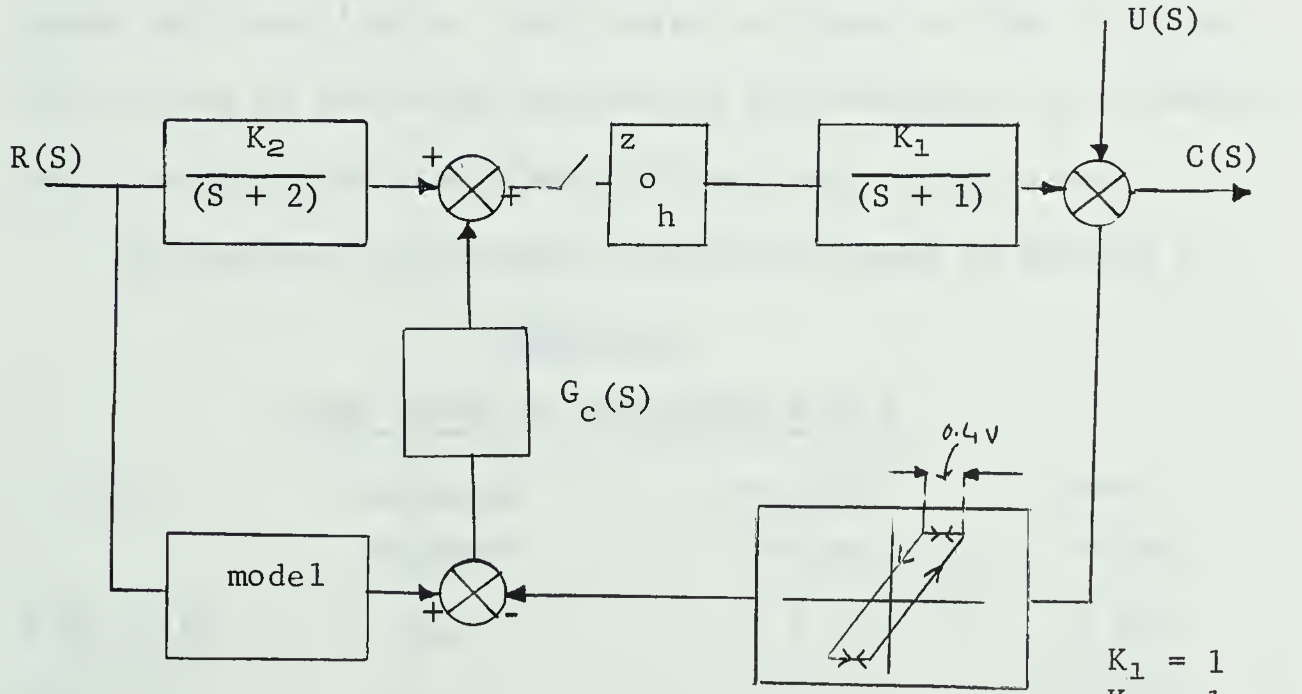


Fig. 3.19

The above system was then simulated on the analog computer to study the response to a step input and also the sensitivity of the

system to parameter variations and external disturbances. The simulation of hysteresis non-linearity is described in appendix B. The effect of hysteresis was, that the output deviated from its normal value by half the value of the hysteresis as shown in fig.3.20.

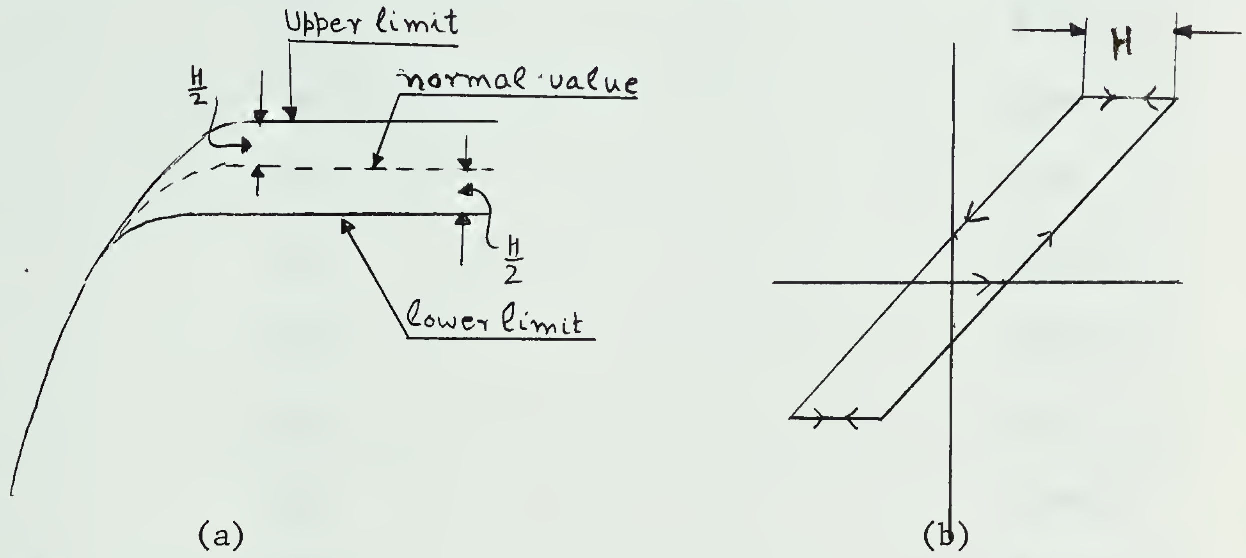


Fig. 3.20

On the computer, the response seemed to drift between its upper and lower limits. The limits are shown in fig. 3.20 (a). This is due to the noise produced in the computer, but in actual practice the output would stay at one limit or the other.

The various disturbances studied are shown in table 3.8.

TABLE 3.8

STEP INPUT TO THE SYSTEM = 10 V

Fig.	Parameter Changed	Original Value	New Value
3.18 - 1&2	K_2	1	1.02
3.18 - 3	K_2	1	1.01
3.18 - 4&5	K_2	1	1.005
3.21 - 1	K_2	1	1.1
3.21 - 2	K_2	1	1.3

Fig.	Parameter Changed	Original Value	New Value
3.21 - 3	K_2	1	1.5
3.21 - 4	K_2	1	0.95
3.21 - 5	K_2	1	0.99
3.22 - 1	K_2	1	0.98
3.22 - 2	K_2	1	0.95
3.22 - 3	K_2	1	0.90
3.22 - 4	K_2	1	0.70
3.22 - 5	K_2	1	0.50
3.23 - 1	K_1	1	1.005
3.23 - 2	K_1	1	1.01
3.23 - 3	K_1	1	1.02
3.23 - 4	K_1	1	1.05
3.24 - 1	K_1	1	1.1
3.24 - 2	K_1	1	1.5
3.25 - 1	K_1	1	0.995
3.25 - 2	K_1	1	0.99
3.25 - 3	K_1	1	0.98
3.25 - 4	K_1	1	0.95
3.26 - 1	K_1	1	0.90
3.26 - 2	K_1	1	0.70
3.26 - 3	K_1	1	0.50
3.26 - 4	Time constant	$\frac{1}{(S + 1)}$	$\frac{1}{(S + 1.01)}$
3.26 - 5	Time constant	$\frac{1}{(S + 1)}$	$\frac{1}{(S + 1.05)}$

Fig.	Parameter Changed	Original Value	New Value
3.27 - 1	Time constant	$\frac{1}{(S + 1)}$	$\frac{1}{(S + 1.11)}$
3.27 - 2	Time constant	$\frac{1}{(S + 1)}$	$\frac{1}{(S + 1.2)}$
3.27 - 3	Time constant	$\frac{1}{(S + 1)}$	$\frac{1}{(S + 1.3)}$
3.27 - 4	Time constant	$\frac{1}{(S + 1)}$	$\frac{1}{(S + 1.5)}$
3.27 - 5	Time constant	$\frac{1}{(S + 1)}$	$\frac{1}{(S + 2)}$
3.28 - 1	Time constant	$\frac{1}{(S + 1)}$	$\frac{1}{(S + 0.99)}$
3.28 - 2	Time constant	$\frac{1}{(S + 1)}$	$\frac{1}{(S + 0.95)}$
3.28 - 3	Time constant	$\frac{1}{(S + 1)}$	$\frac{1}{(S + 0.9)}$
3.28 - 4	Time constant	$\frac{1}{(S + 1)}$	$\frac{1}{(S + 0.7)}$

When the plant was changed to $\frac{1}{(S + 0.5)}$, the output became very oscillatory. This is due to the fact that the controller had been designed for a plant $\frac{1}{(S + 1)}$. If there was a large veriation in the plant, the controller did not act effectively.

The above observations show that any disturbance which is able to produce a signal at the output of the hysteresis will soon be detected by the controller and its effect on the output removed.

A ramp input was then applied to the system. The response was good as long as there was no variation in parameter. A parameter variation would give rise to a ramp disturbance at the output in

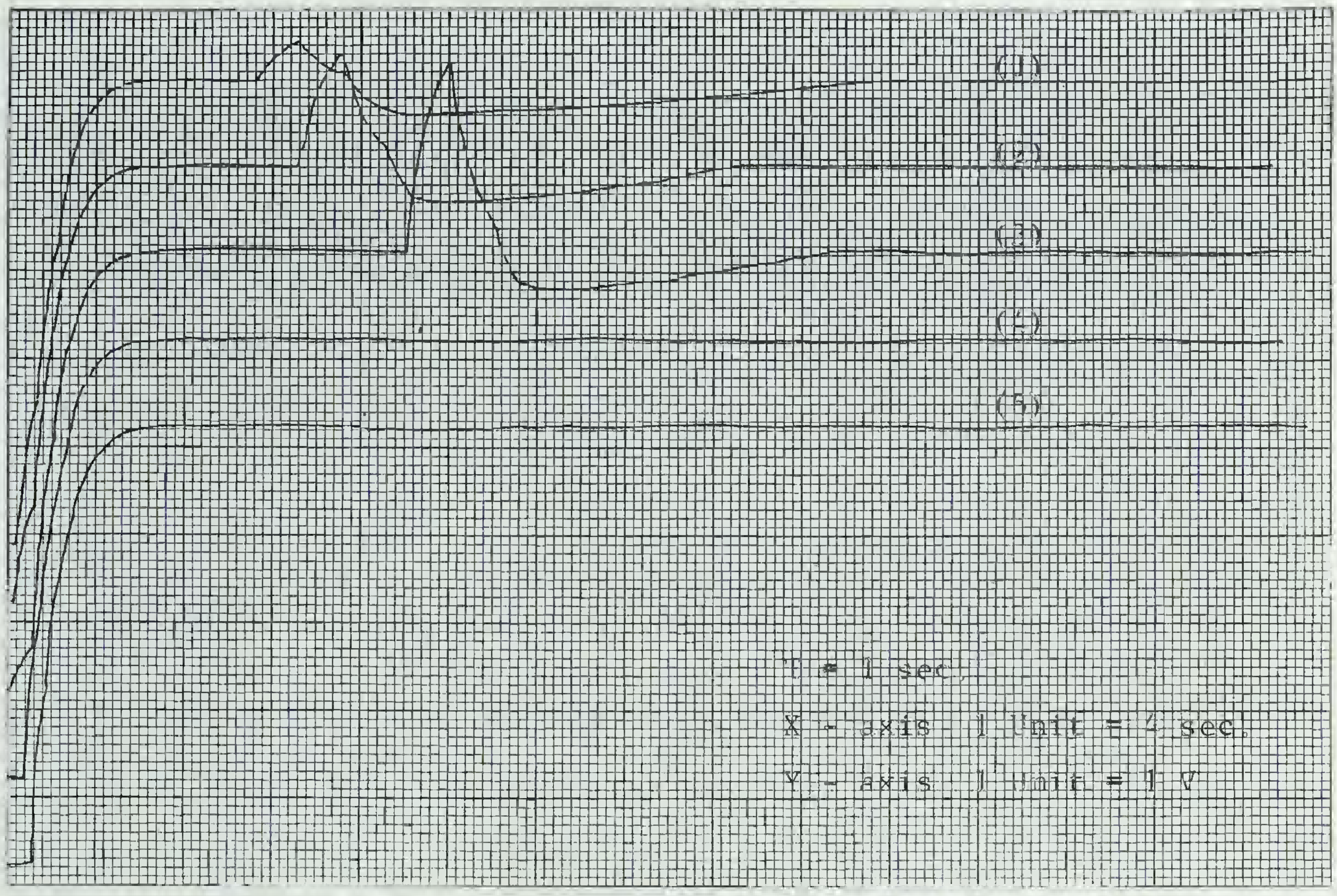


Fig. 2.21

Fig. 2.22

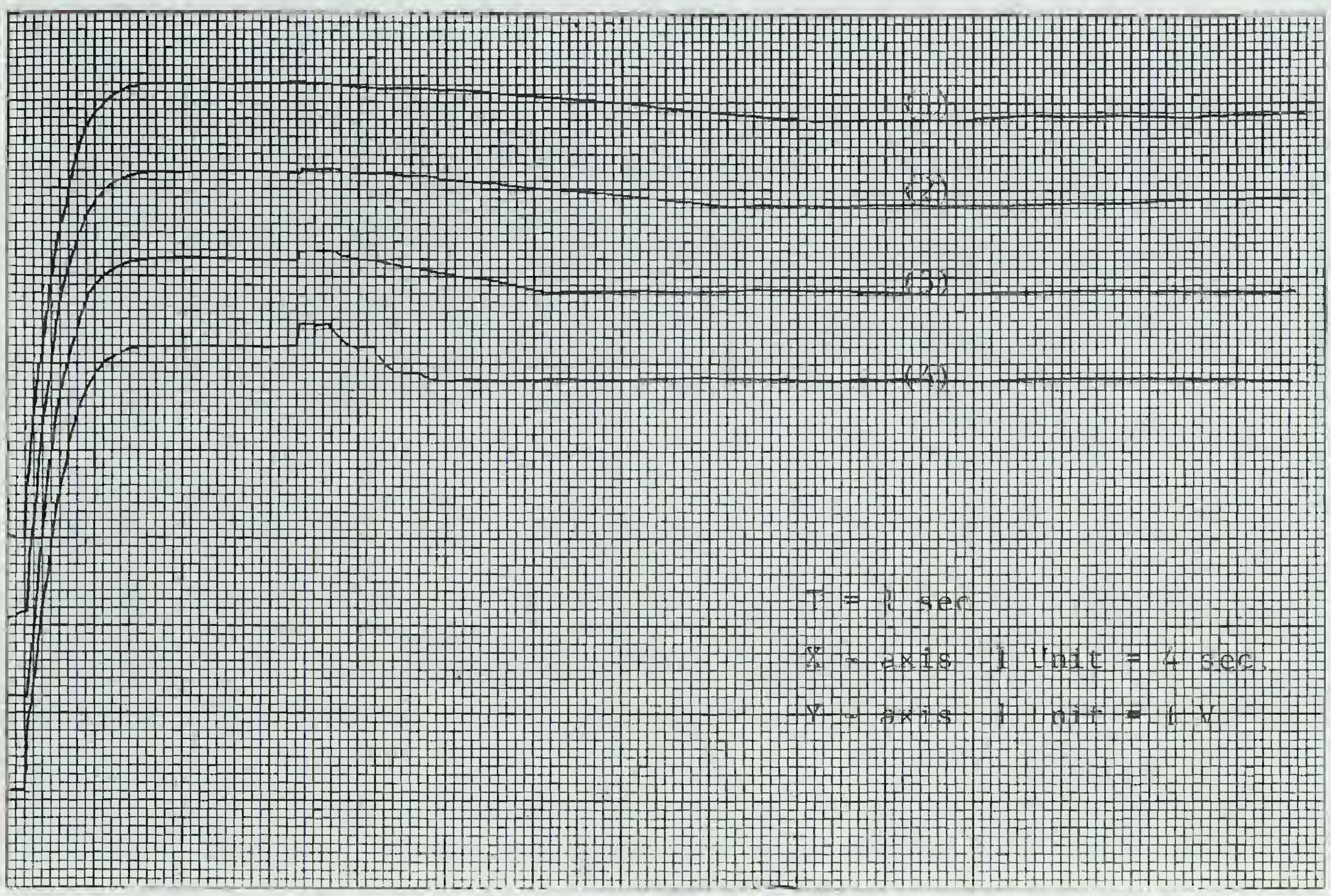


Fig. 3.23

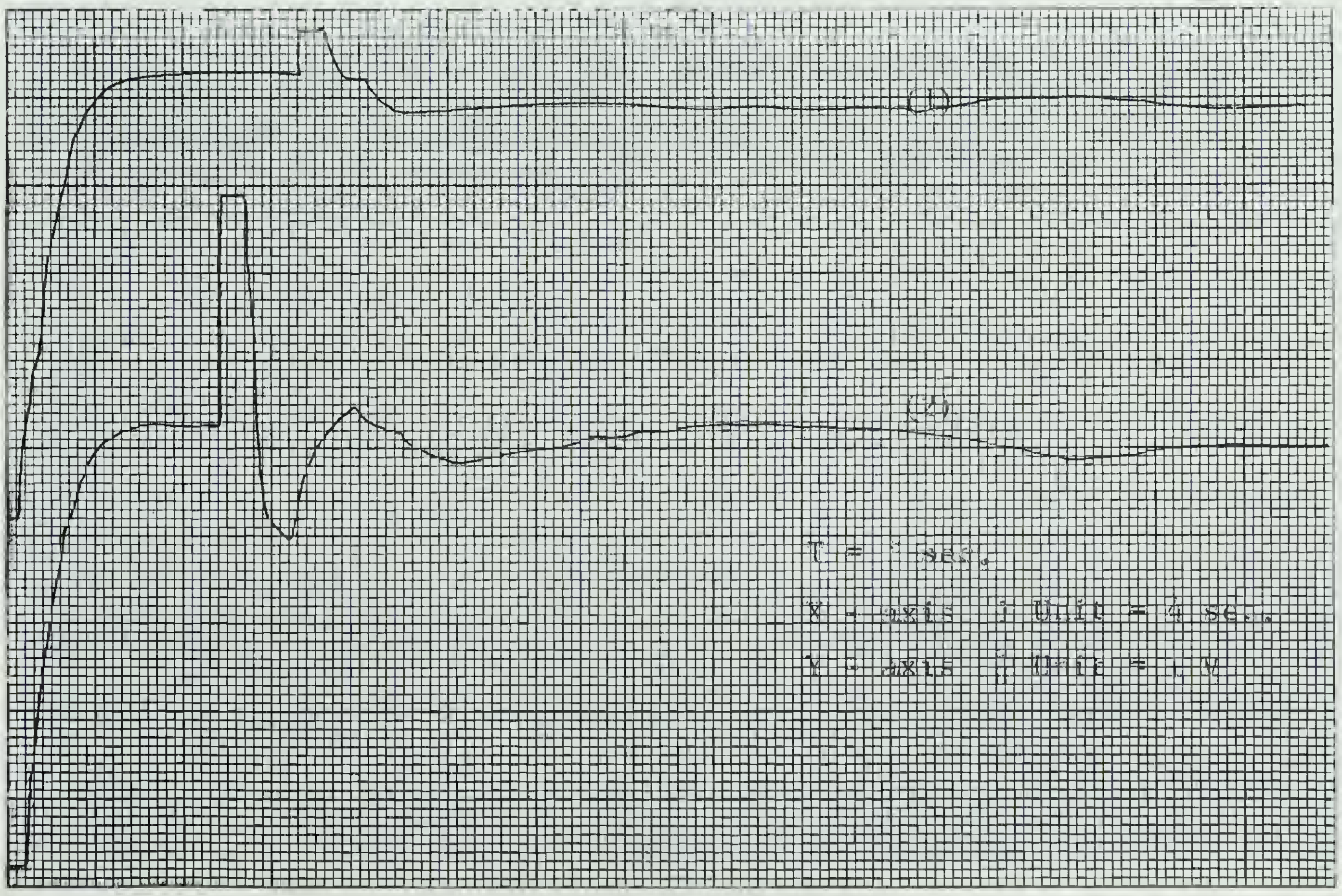


Fig. 3.24

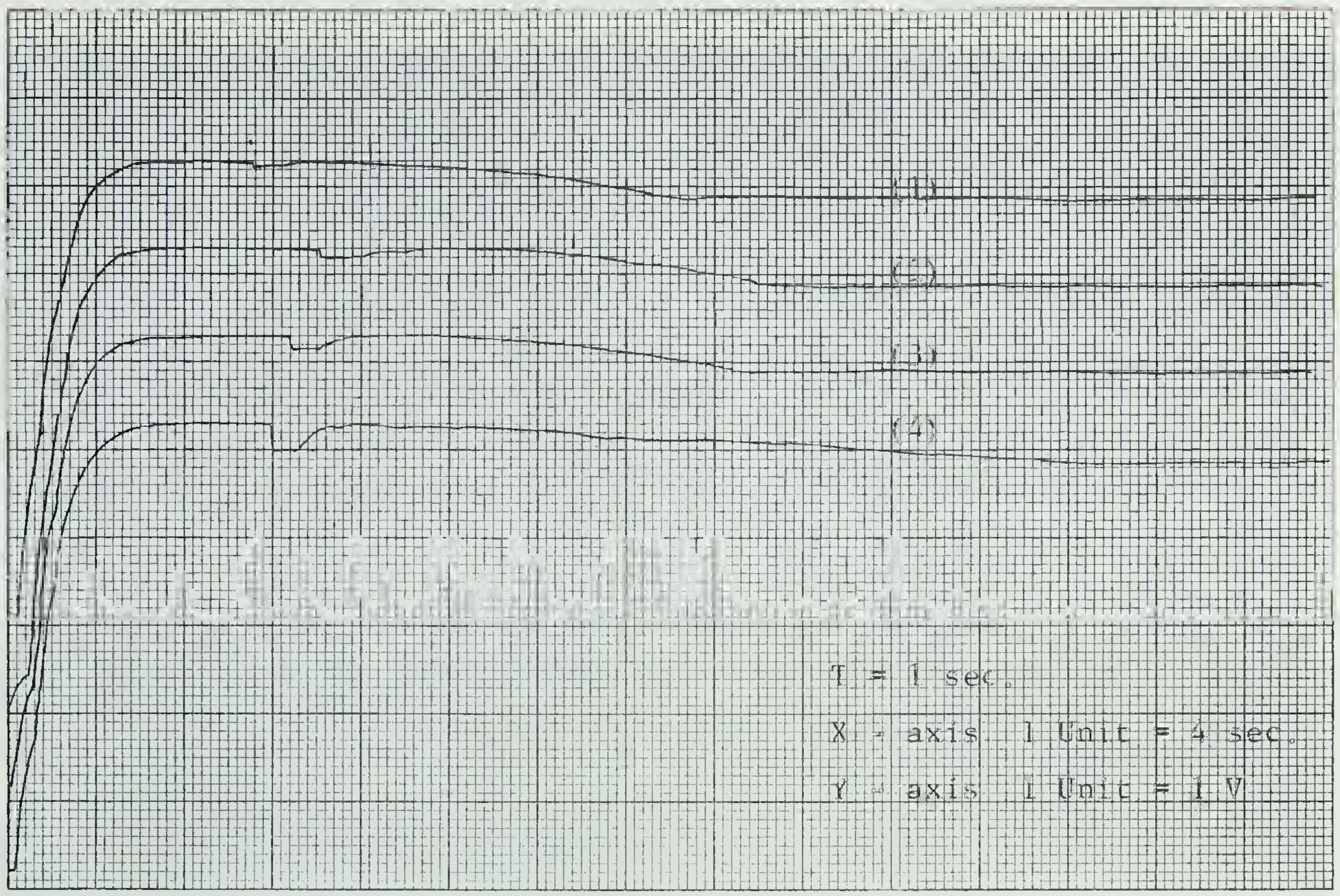
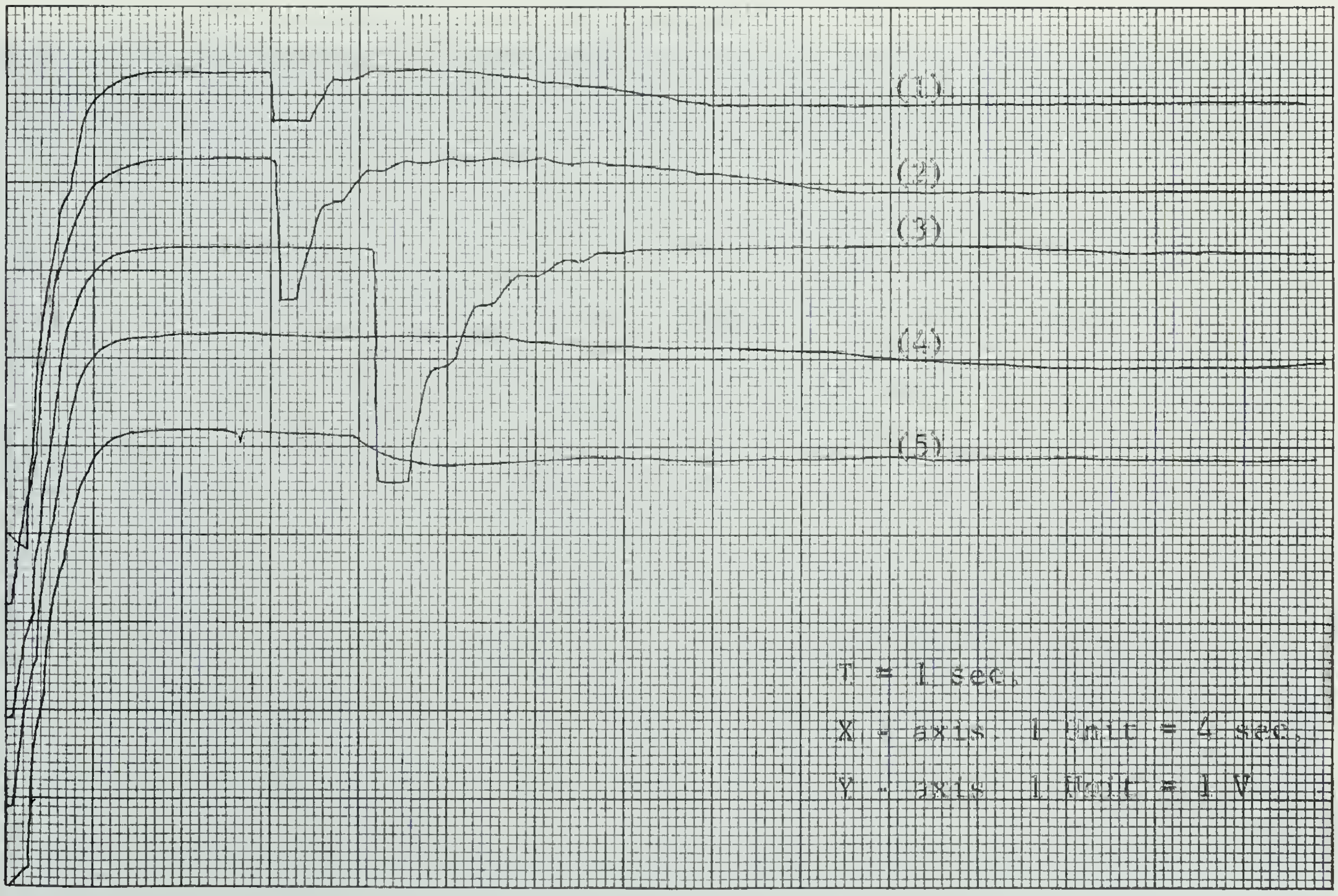

(1)
(2)
(3)
(4)
T = 1 sec.
X - axis 1 Unit = 4 sec.
Y - axis 1 Unit = 1 V

Fig. 3.25


(1)
(2)
(3)
(4)
(5)
T = 1 sec.
X - axis 1 Unit = 4 sec.
Y - axis 1 Unit = 1 V

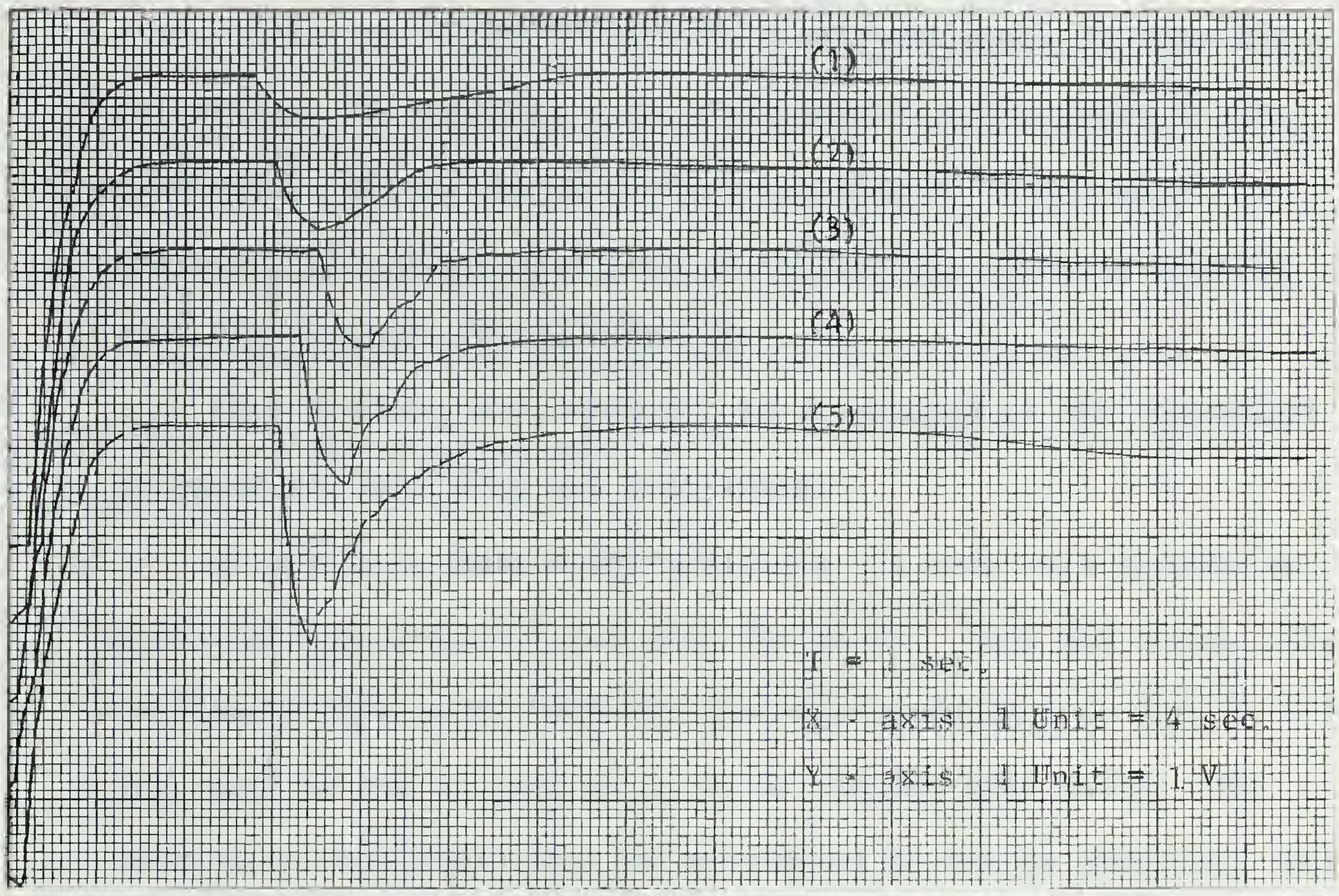


Fig. 3.27

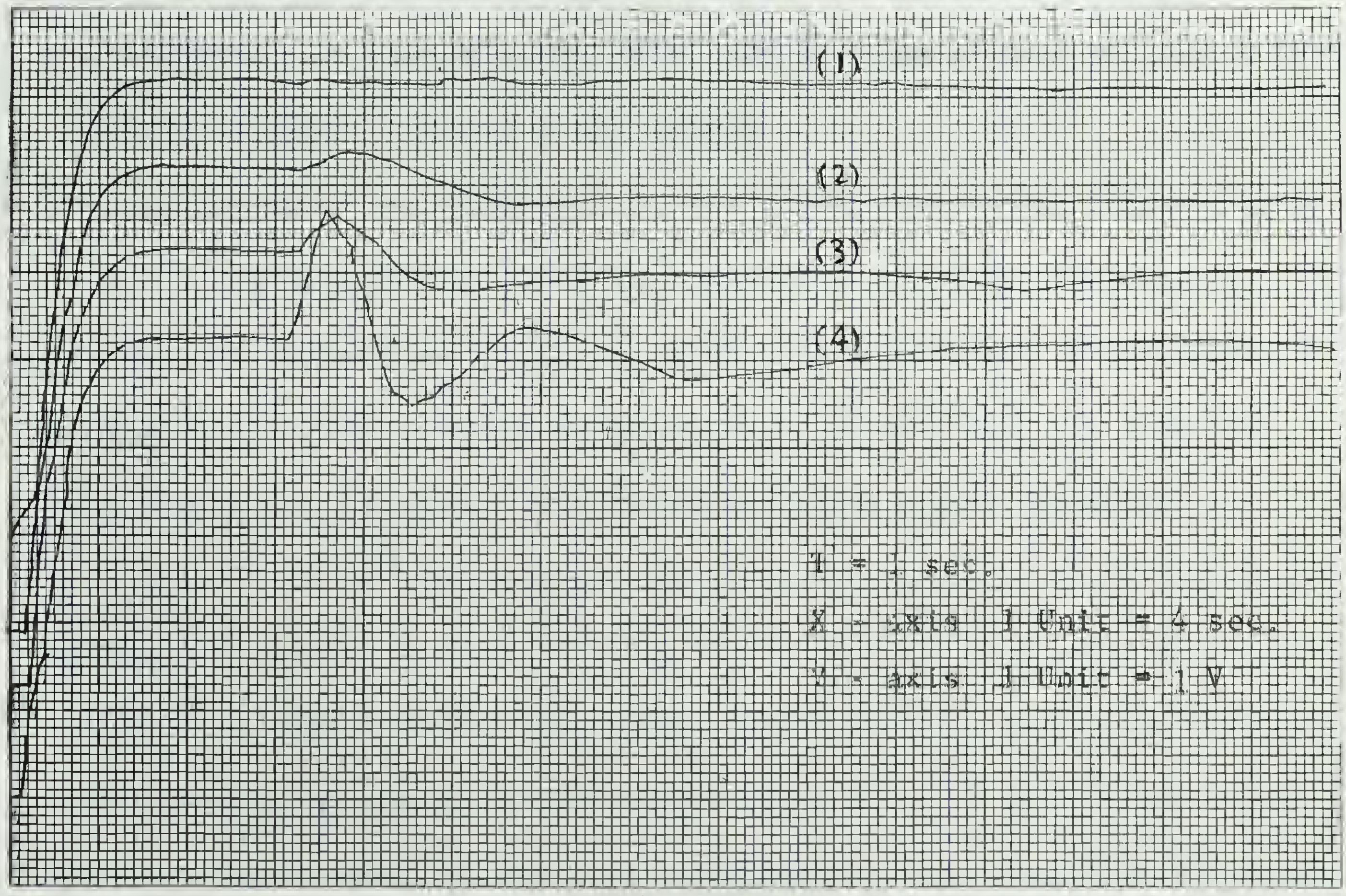


Fig. 3.28

the steady state. The controller was designed for step disturbances only. This controller would give a constant error to ramp disturbances. The constant error will appear at the input to the controller. Since the controller contains a free digital integrator, its output will keep on increasing. This is exactly what was observed on the computer.

This drawback can, however, be eliminated if the controller is designed for both ramp and step inputs.

A continuous data model could also be used here because no non-linearity is associated with the model. This was tried. The continuous data model used was the same as derived in article 2.1. The response was as good as with the sampled data model. The continuous data model will be economical to use in practice. But the use of a continuous data model will be possible when the non-linearity is present only after the sensing device and the forward path transfer functions are linear.

CHAPTER IV

The conditional feedback can also be used to optimise the response of oscillatory systems. The dead-beat controller is designed as usual depending upon the types of disturbances expected. The model is then modified so that the response of the oscillatory system is improved.

A second order oscillatory system has a transfer function of the form $\frac{\omega_n^2}{S^2 + 2\zeta\omega_n S + \omega_n^2}$ $\zeta < 1$. The conditional feedback arrangement for this is shown in fig. 4.1.

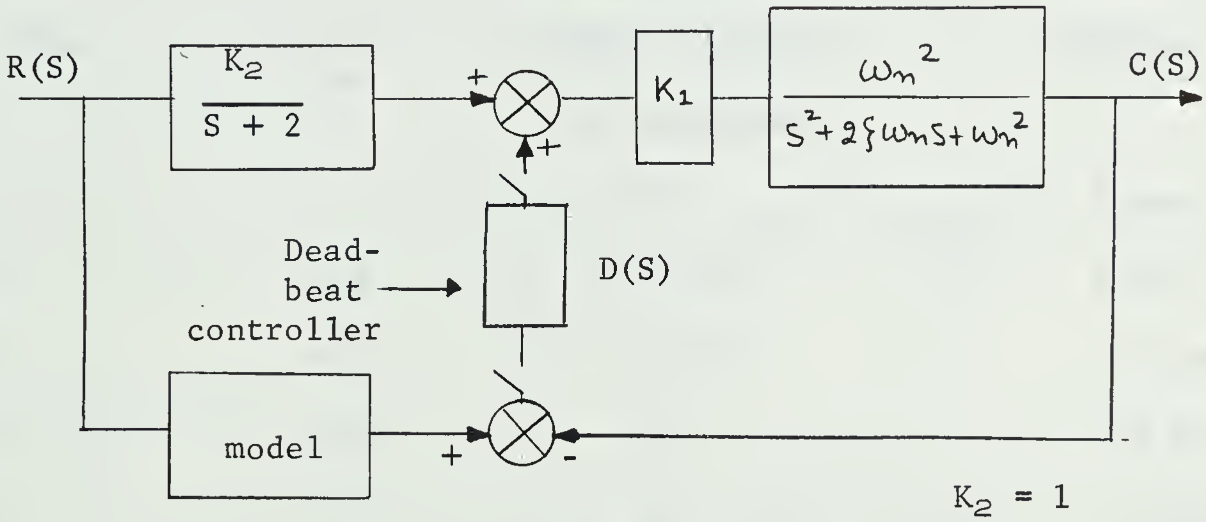


Fig. 4.1

The above system was simulated on the analog computer. It was found that the response of the oscillatory system could be improved by setting the damping ratio of the model to a different value from that of the plant. The optimum response for a plant with the transfer function $\frac{1}{S^2 + 1.2S + 1}$ was found by trial. This is shown in table 4.1

ζ_p = damping ratio of the plant

ζ_m = damping ratio of the model

Settling time refers to the time required for the response to decrease

to and stay within 2% of its final value.

STEP INPUT TO THE SYSTEM = 10 V

Sampling period $T = 0.5$ sec.

Dead-beat controller found by rep-op method

$$D(Z) = 1.931 \frac{1 - 0.1234Z^{-1} - 1.243Z^{-2} + 0.7335Z^{-3}}{1 - 0.425Z^{-1} - 0.4Z^{-2} - 0.175Z^{-3}}$$

$\zeta_p = 0.6$

TABLE 4.1

Fig.	ζ_m	Max. overshoot or undershoot	Settling time
4.2 - 1	1	1%	9 sec.
4.2 - 2	0.8	1.5%	7 sec.
4.2 - 3	0.7	2.5%	3.6 sec.
4.2 - 4	0.75	2%	4.4 sec.
4.2 - 5	0.72	1%	3.6 sec.
4.3 - 1	0.73	1%	3.8 sec.
4.3 - 2&3	0.74	2%	4 sec.
4.3 - 4	0.75	2%	4.4 sec.
4.3 - 5	0.72	1%	3.6 sec.

The above data indicates an optimum value of ζ_m to be 0.72.

The effect of sampling period of the controller on the optimum response was then studied. It was found that as the sampling period is reduced, the response improves. The disadvantage of reducing the time period is that the controller is required to give larger steps. If these large steps saturate the plant, it will not be possible to

reduce the sampling period beyond a certain point. The effects of variation of sampling time period on the response are shown in table 4.2. All dead-beat controllers were designed by the rep-op method.

1. Transfer function of plant = $\frac{1}{S^2 + S + 1}$

$\zeta_p = 0.5$

T = 1 sec.

$$D(Z) = 1.852 \frac{1 - 0.861Z^{-1} + 0.394Z^{-2}}{1 - 0.6Z^{-1} - 0.4Z^{-2}}$$

T = 0.5 sec.

$$D(Z) = 2.333 \frac{1 - 0.1043Z^{-1} - 1.293Z^{-2} + 0.812Z^{-3}}{1 - 0.234Z^{-1} - 0.516Z^{-2} - 0.25Z^{-3}}$$

T = 0.2 sec.

$$D(Z) = 13.94 \frac{1 - 0.7730Z^{-1} - 1.115Z^{-2} + 0.899Z^{-3}}{1 - 0.25Z^{-1} - 0.5Z^{-2} - 0.25Z^{-3}}$$

2. Transfer function of plant = $\frac{1}{S^2 + 0.6S + 1}$

$\zeta_p = 0.3$

T = 1 sec.

$$D(Z) = 1.523 \frac{1 - 0.9175Z^{-1} + 0.5775Z^{-2}}{1 - 0.5625Z^{-1} - 0.4375Z^{-2}}$$

T = 0.5 sec.

$$D(Z) = 2.035 \frac{1 - 0.1124Z^{-1} - 1.484Z^{-2} + 1.079Z^{-3}}{1 - 0.25Z^{-1} - 0.5Z^{-2} - 0.25Z^{-3}}$$

T = 0.2 sec.

$$D(Z) = 13.23 \frac{1 - 0.7723Z^{-1} - 1.175Z^{-2} - Z^{-3}}{1 - 0.25Z^{-1} - 0.5Z^{-2} - 0.25Z^{-3}}$$

TABLE 4.2

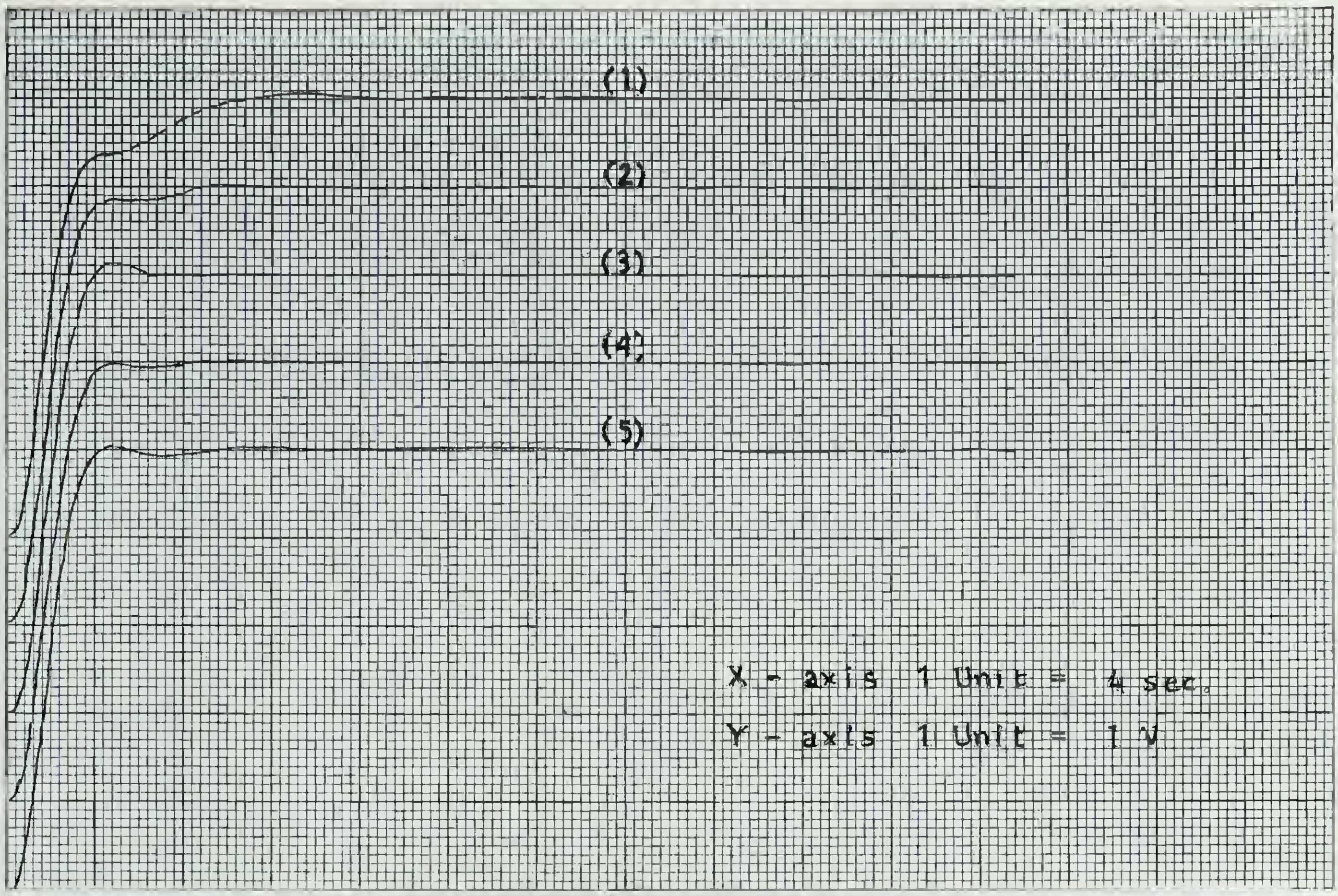
Fig.	ζ_p	T	Max. overshoot	Settling time	ζ_m
*4.4 - 1	0.5	-	14%	8 sec.	-
4.4 - 2	0.5	1 sec.	4%	7.8 sec.	0.663
4.4 - 3	0.5	0.5 sec.	2%	6.4 sec.	0.667
4.4 - 4	0.5	0.2 sec.	1%	3.8 sec.	0.73
*4.4 - 5	0.3	-	33%	12 sec.	-
4.4 - 6	0.3	1 sec.	16%	11.2 sec.	0.555
4.4 - 7	0.3	0.5 sec.	12%	10.8 sec.	0.55
4.4 - 8	0.3	0.2 sec.	3%	8 sec.	0.6375

The effect of change of parameters on the output were then

---

* These curves show the response of the transfer function

$\dfrac{\omega_n^2}{(S+2)(S^2+2\zeta\omega_n S+\omega_n^2)}$ to a step input, i.e. the model and the controller are out of circuit. The other curves show the improvement of this response by the use of conditional feedback.


(1)
(2)
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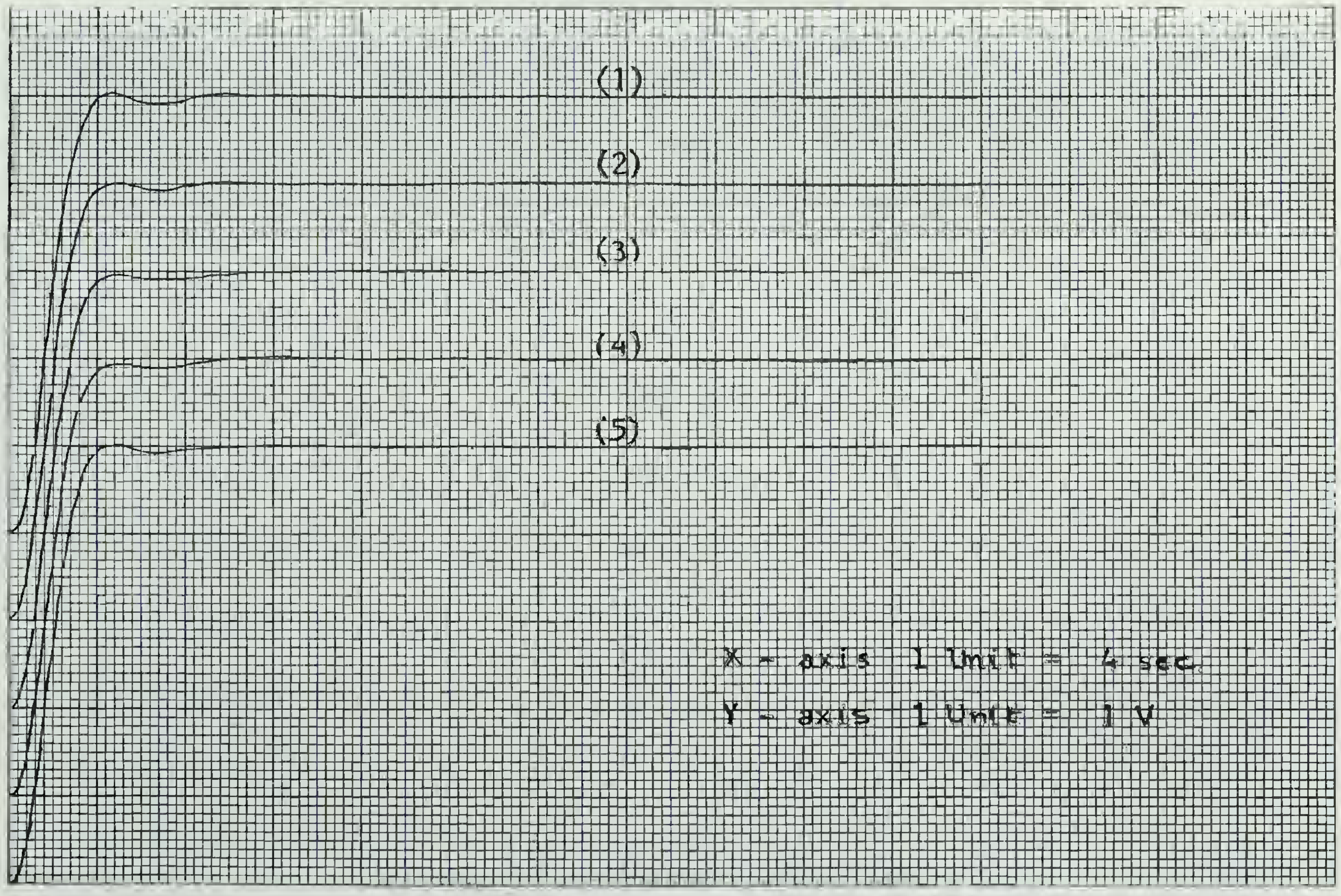
(5)
X - axis 1 Unit = 4 sec.
Y - axis 1 Unit = 1 V

Fig. 4.2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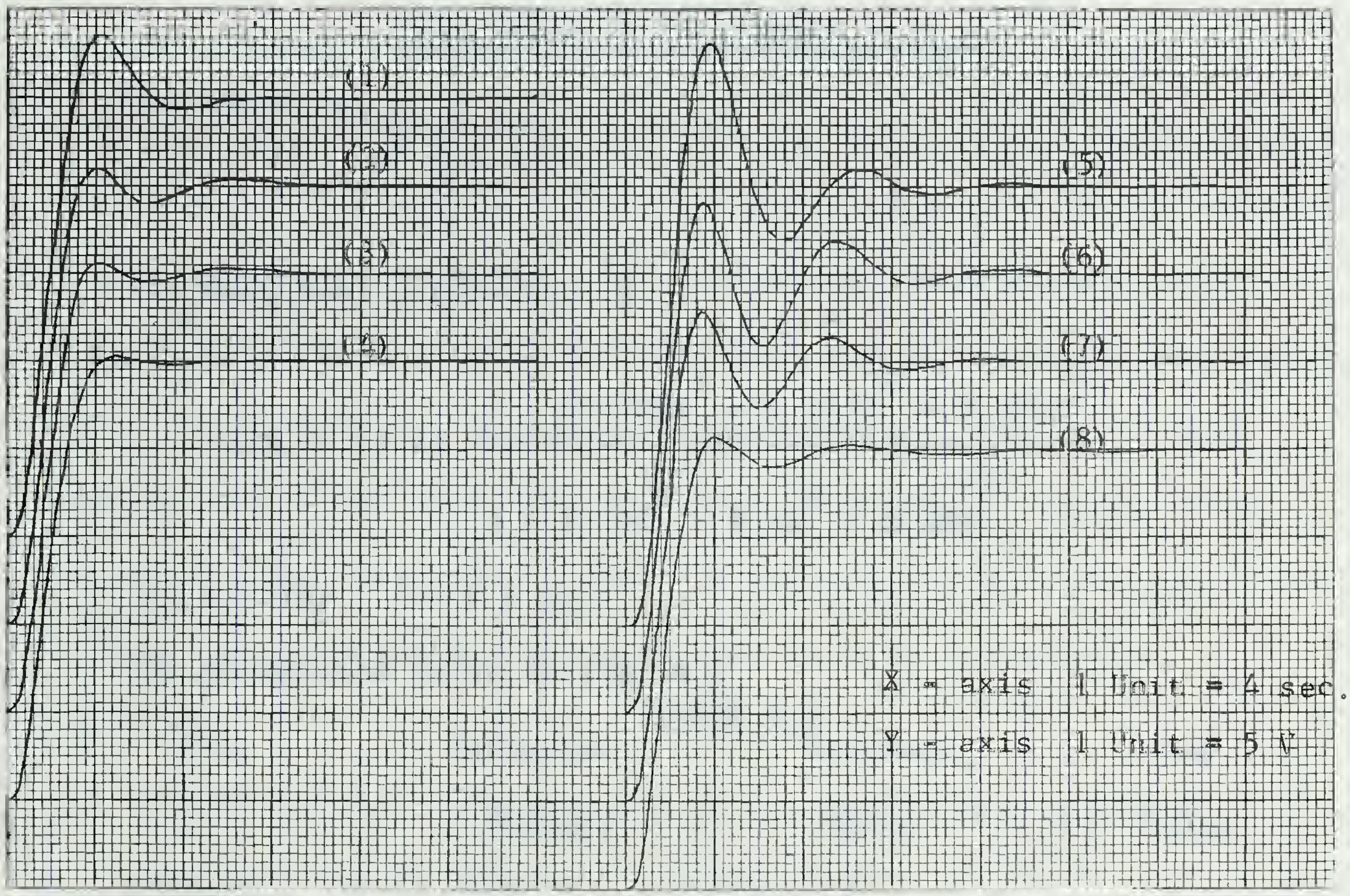
(4)
(5)
X - axis 1 Unit = 4 sec.
Y - axis 1 Unit = 1 V

Fig. 4.3


(1)
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(6)
(7)
(8)
X - axis 1 Unit = 4 sec.
Y - axis 1 Unit = 5 V

Fig. 4.4

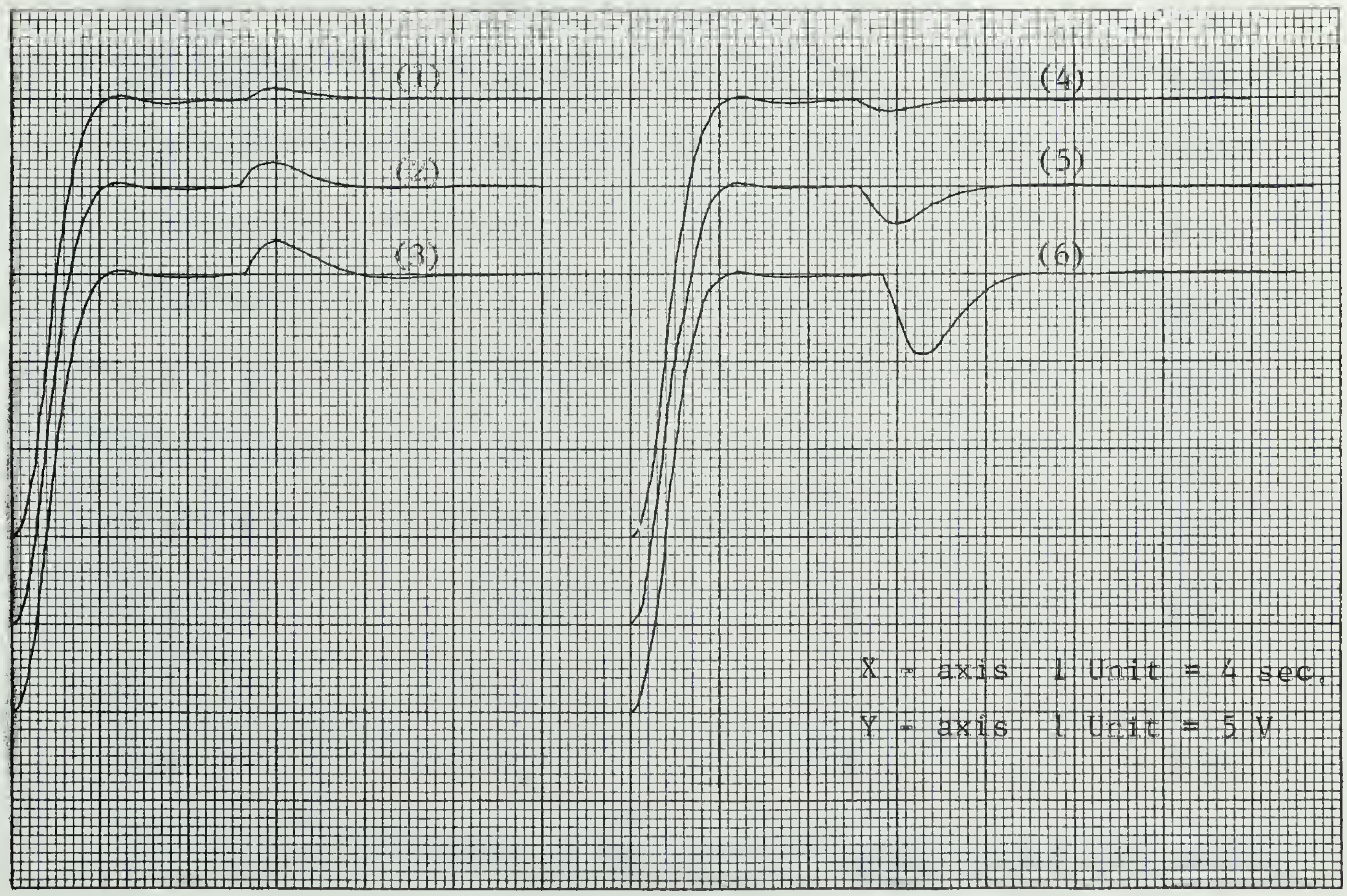

(1)
(2)
(3)
(4)
(5)
(6)
X - axis 1 Unit = 4 sec.
Y - axis 1 Unit = 5 V

Fig. 4.5

studied. The results are shown in table 4.3.

STEP INPUT TO THE SYSTEM = 50 V

The plant is $\dfrac{1}{S^2 + S + 1}$

The dead-beat controller is

$$D(Z) = 13.94 \, \frac{1 - 0.7730Z^{-1} - 1.115Z^{-2} + 0.899Z^{-3}}{1 - 0.25Z^{-1} - 0.5Z^{-2} - 0.25Z^{-3}}$$

$\zeta_m = 0.73.$ T= 0.2 sec.

The parameters refer to fig. 4.1

TABLE 4.3

Fig.	Parameter Changed	Original Value	New Value
4.5 - 1	K_1	1	1.1
4.5 - 2	K_1	1	1.3
4.5 - 3	K_1	1	1.5
4.5 - 4	K_1	1	0.9
4.5 - 5	K_1	1	0.7
4.5 - 6	K_1	1	0.5
4.6 - 1	K_2	1	1.1
4.6 - 2	K_2	1	1.3
4.6 - 3	K_2	1	1.5
4.6 - 4	K_2	1	0.9
4.6 - 5	K_2	1	0.7
4.6 - 6	K_2	1	0.5

The effects of change of the natural frequency and the damping ratio of the plant on the transient response are shown in tables

4.4 and 4.5.

The optimum response from the oscillatory plant can also be achieved by a dead-beat controller and the conventional feedback. The conventional feedback will, in fact, be economical to use because it does not require a model. The conditional feedback was, therefore, compared with conventional feedback and it was found that the former is superior to the latter in performance. The conditional feedback system was much less sensitive to parameter changes then the conventional feedback system. In addition to this the conditional feedback system has other advantages which were discussed in chapter I.

The two systems were compared on the following two bases:

1. Same sampling period for the controllers.
2. Same rise time of the response to a step input.

The comparison was made on the basis of transient response.

In the first case (same sampling period for the controllers), the response to a disturbance will be similar for both types of feedback, because the feedback loop for disturbances is essentially the same. The response to disturbances will be better for conditional feedback in the second case (same rise time to step inputs). The disturbances will be removed much quicker and with less overshoot, because of the shorter sampling period. The arrangement of the two types of systems is shown in fig. 4.7.

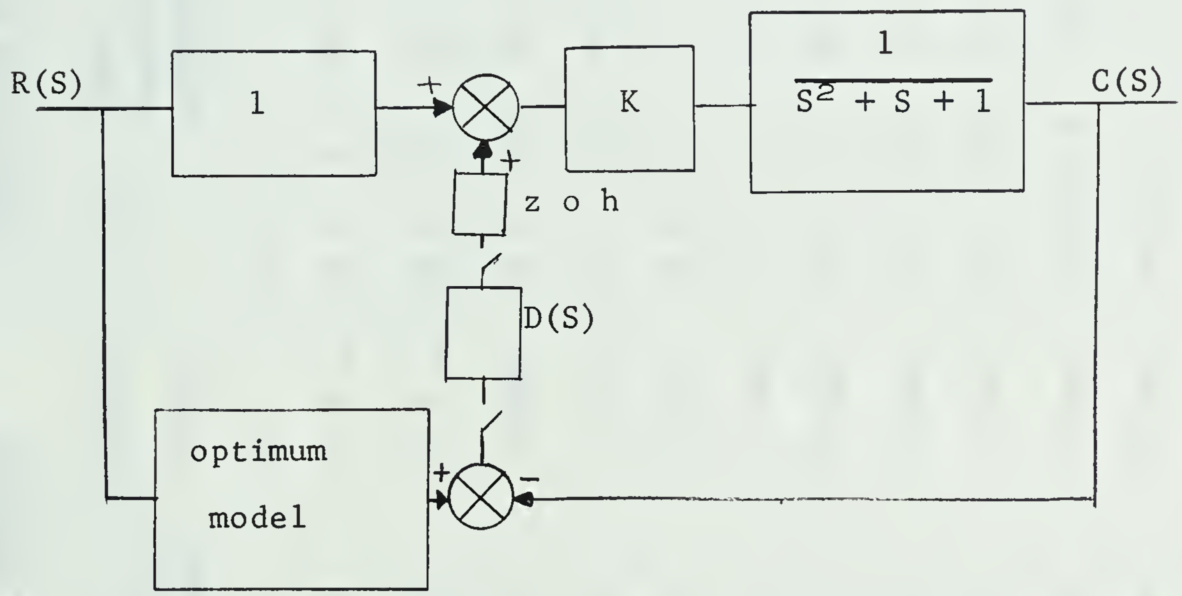
1. <u>The same sampling period for the controllers</u>

Sampling period = 0.2 sec.

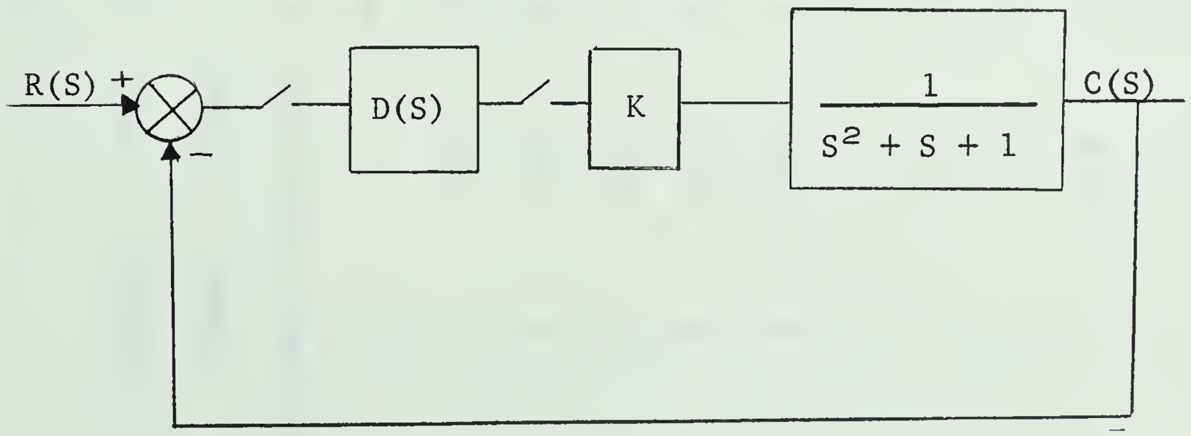
$$D(Z) = 14.25 \frac{1 - 0.7988Z^{-1} - 0.9270Z^{-2} + 0.798Z^{-3}}{1 - 0.25Z^{-1} - 0.5Z^{-2} - 0.25Z^{-3}}$$

This controller was used for both the types of feedback systems.

STEP INPUT TO THE SYSTEM = 50 V



(a) Conditional feedback



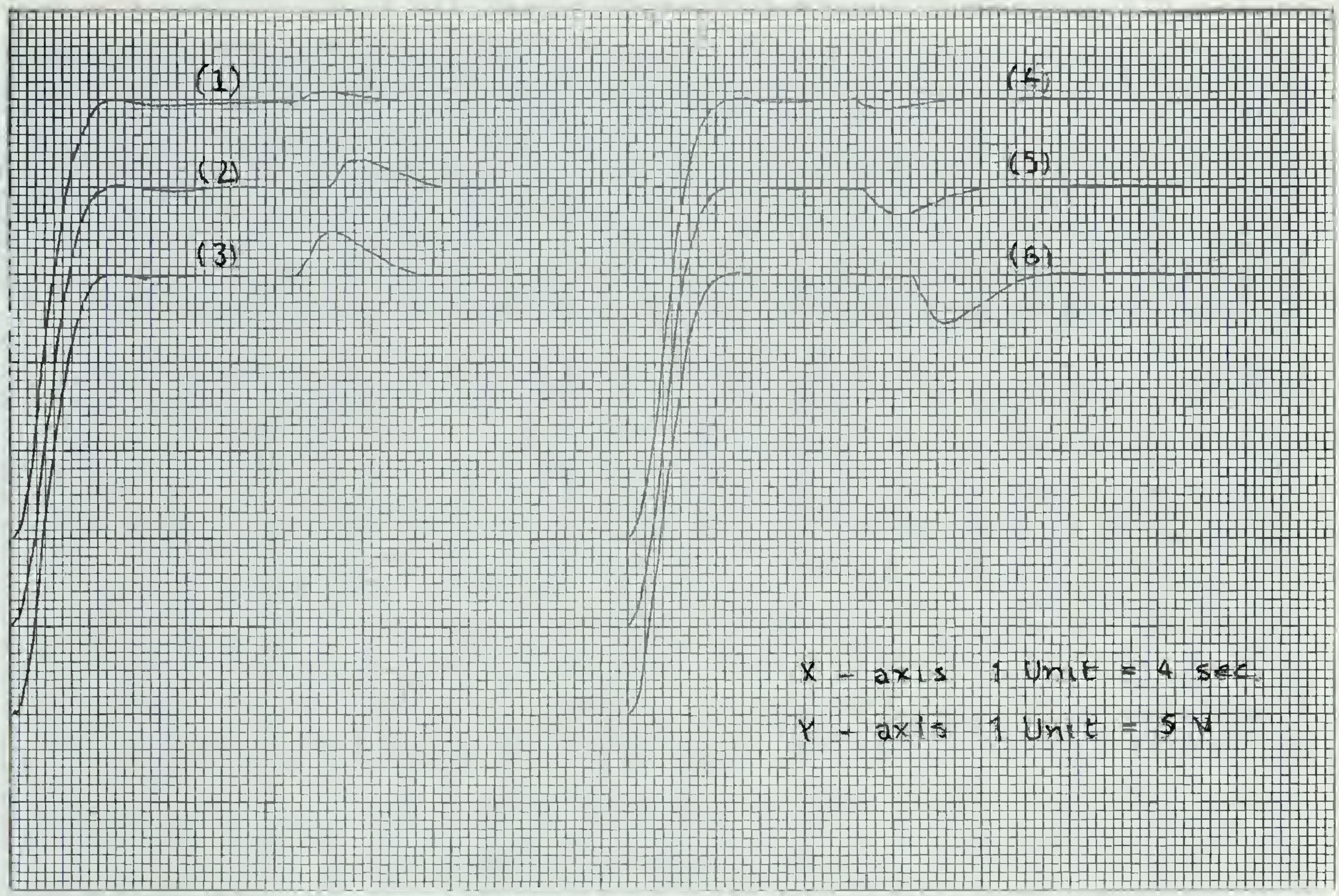
(b) Conventional feedback

Fig. 4.7

TABLE 4.4

Parameter Changed	Original Value	New Value	Conventional feedback: Fig.	Max. overshoot/ undershoot	Settling time	Conditional feedback: Fig.	Max. overshoot/ undershoot	Settling time
Response without disturbance			4.8 - 1	-	0.6 sec.	4.8 - 2	-	4 sec.
K	1	1.1	4.8 - 3	7%	1.2 sec.	4.8 - 4	0%	4 sec.
K	1	1.3	4.8 - 5	19%	1.6 sec.	4.8 - 6	0%	4 sec.
K	1	1.5	4.8 - 7	31%	2 sec.	4.8 - 8	0%	4 sec.
K	1	0.9	4.8 - 9	0%	1.2 sec.	4.8 - 10	0%	4 sec.
K	1	0.7	4.9 - 1	0%	2 sec.	4.9 - 2	0%	4.4 sec.
K	1	0.5	4.9 - 3	0%	4 sec.	4.9 - 4	0%	5.6 sec.
ω_n^2	1	1.1	4.9 - 5	6%	4 sec.	4.9 - 6	0%	5.6 sec.
ω_n^2	1	1.3	4.9 - 7	18%	4 sec.	4.9 - 8	1%	5.6 sec.
ω_n^2	1	1.5	4.9 - 9	29%	4 sec.	4.9 - 10	1%	5.6 sec.
ω_n^2	1	0.9	4.10 - 1	3%	4 sec.	4.10 - 2	0.5%	4 sec.
ω_n^2	1	0.7	4.10 - 3	8%	8 sec.	4.10 - 4	4%	8 sec.
ω_n^2	1	0.5	4.10 - 5	18%	12 sec.	4.10 - 6	12%	8.4 sec.
ζ_p	0.5	0.55	4.10 - 7	2%	5.2 sec.	4.10 - 8	0.5%	4 sec.

Parameter Changed	Original Value	New Value	Conventional feedback: Fig.	Conventional feedback: Max. overshoot/ undershoot	Conventional feedback: Settling time	Conditional feedback: Fig.	Conditional feedback: Max. overshoot/ undershoot	Conditional feedback: Settling time
ζ_p	0.5	0.65	4.10 - 9	4%	5.6 sec.	4.10 - 10	2%	6.4 sec.
ζ_p	0.5	0.75	4.11 - 1	5%	6 sec.	4.11 - 2	3%	7.2 sec.
ζ_p	0.5	0.45	4.11 - 3	2.5%	4.8 sec.	4.11 - 4	0%	6.4 sec.
ζ_p	0.5	0.35	4.11 - 5	8%	5.2 sec.	4.11 - 6	1%	6.4 sec.
ζ_p	0.5	0.25	4.11 - 7	14%	8 sec.	4.11 - 8	1.5%	9.6 sec.
A step disturbance of 5 V is applied at the output			4.11 - 9	-	0.6 sec.	4.11 - 10	-	0.6 sec.


(1)
(2)
(3)
(4)
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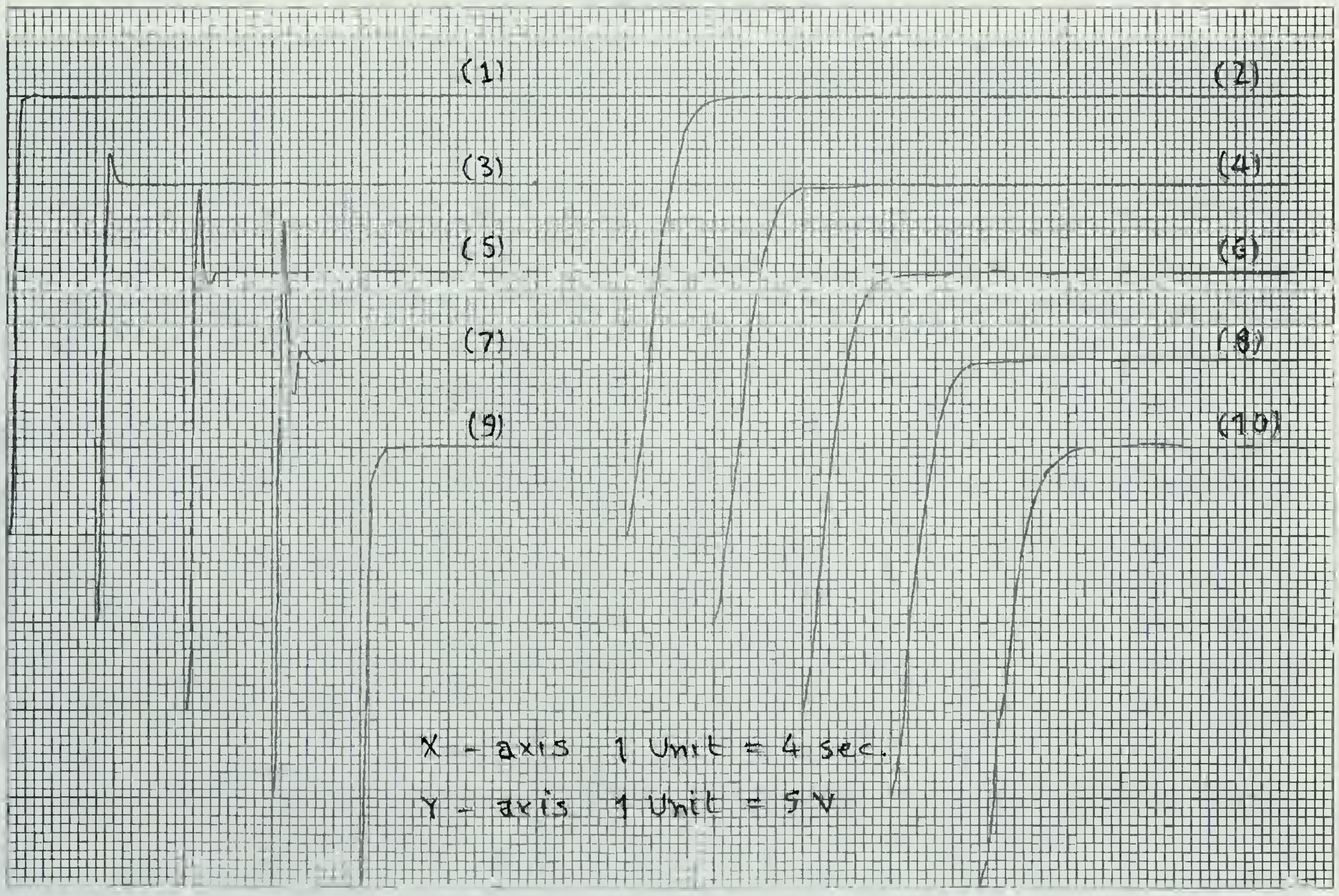
(6)
X - axis 1 Unit = 4 sec.
Y - axis 1 Unit = 5 V

Fig. 4.6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(9)
(10)
X - axis 1 Unit = 4 sec.
Y - axis 1 Unit = 5 V

Fig. 4.8

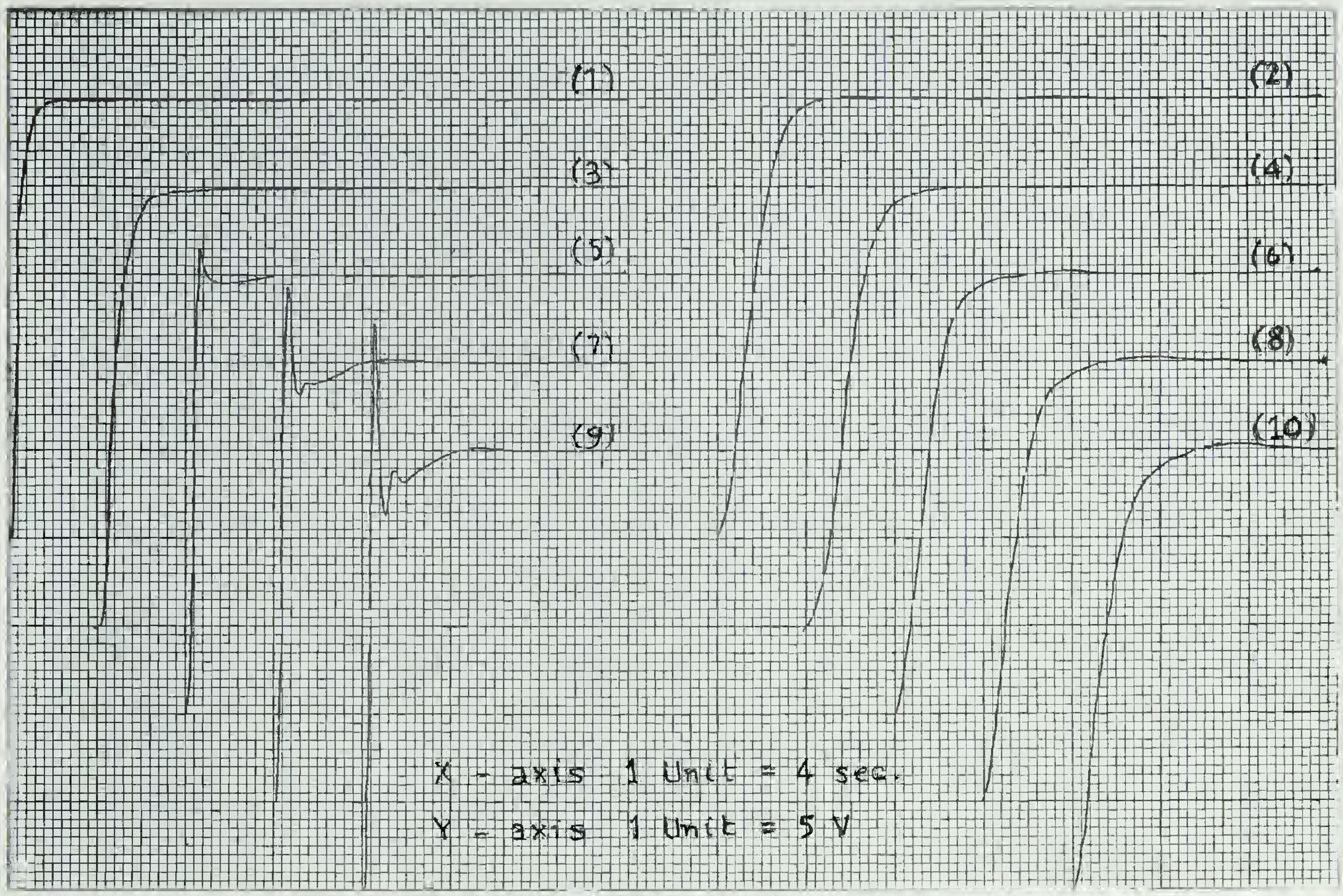


Fig. 4.9

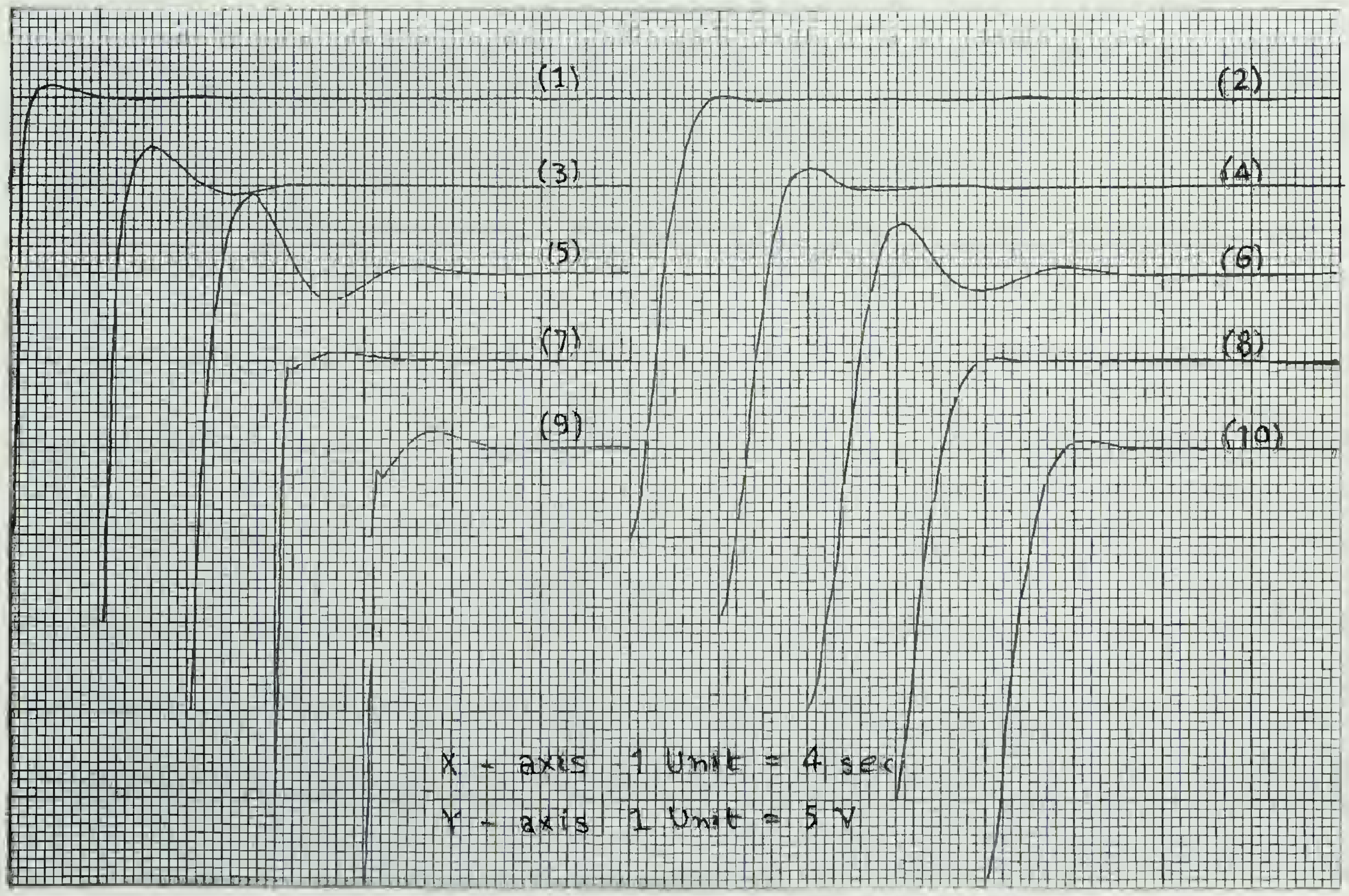


Fig. 4.10

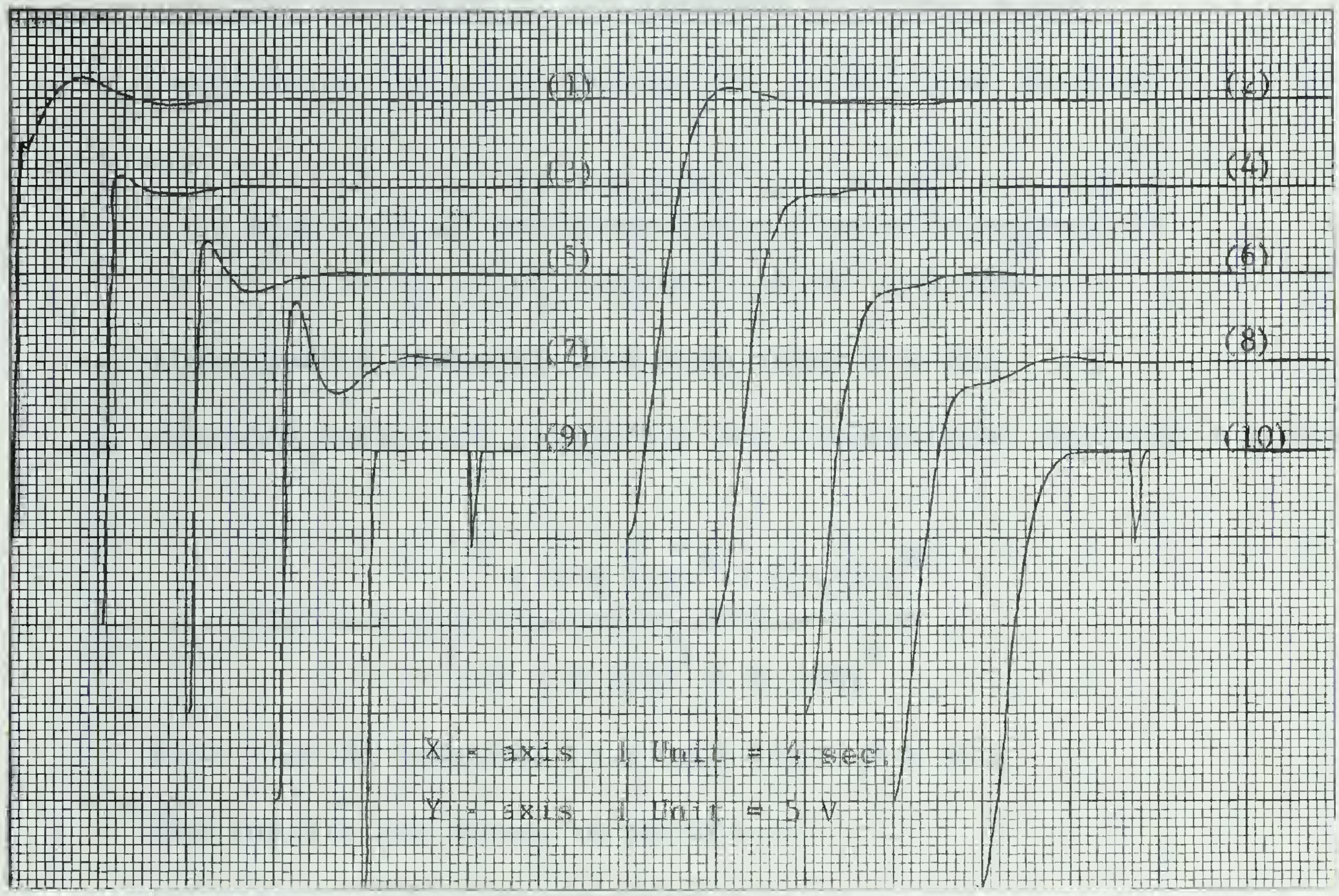


Fig. 4.11

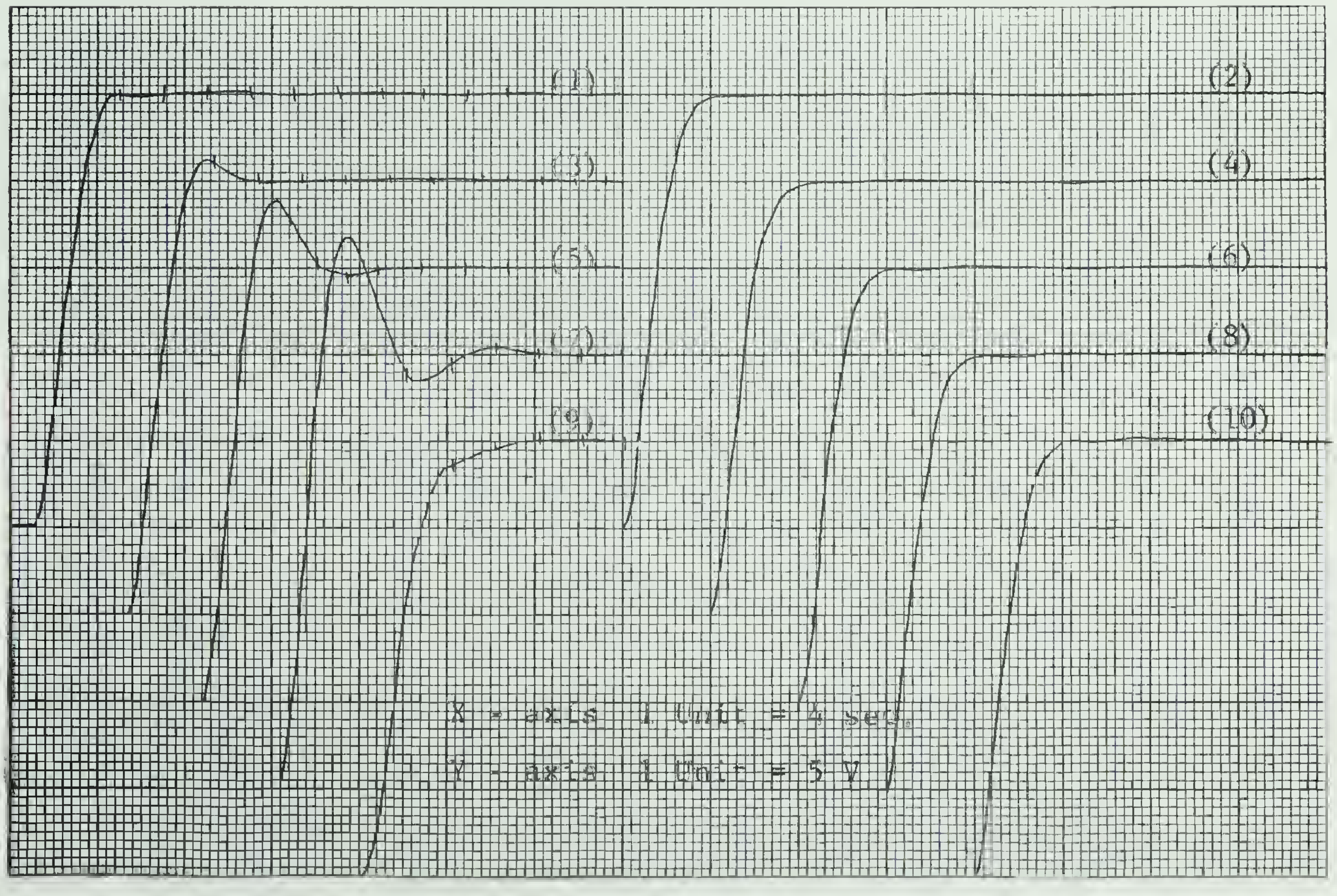


Fig. 4.12

2. <u>The same rise time of the response to a step input</u>

Sampling period of controller for conditional feedback = 0.2 sec.

$$D(Z) = 14.25 \frac{(1 - 0.7988Z^{-1} - 0.9270Z^{-2} + 0.798Z^{-3})}{(1 - 0.25Z^{-1} - 0.5Z^{-2} - 0.25Z^{-3})}$$

Rise time of response to a step input = 4 sec.

Sampling period of controller for conventional feedback = 1.33 sec.

$$D(Z) = 0.813 \frac{1 + 0.113Z^{-1} + 0.1395Z^{-2}}{1 - 0.662Z^{-1} - 0.338Z^{-2}}$$

Rise time of response to a step input = 4 sec.

The results are shown in table 4.5

Step input to the system = 50 V

TABLE 4.5

Parameter Changed	Original Value	New Value	Conventional feedback: Fig.	Conventional feedback: Max. overshoot/ undershoot	Conventional feedback: Settling time	Conditional feedback: Fig.	Conditional feedback: Max. overshoot	Conditional feedback: Settling time
Response without disturbance			4.12 - 1	0%	4 sec.	4.12 - 2	0%	4 sec.
K	1	1.1	4.12 - 3	5%	6 sec.	4.12 - 4	0%	4 sec.
K	1	1.3	4.12 - 5	15%	8.4 sec.	4.12 - 6	0%	4 sec.
K	1	1.5	4.12 - 7	27%	11.6 sec.	4.12 - 8	0%	4 sec.
K	1	0.9	4.12 - 9	0%	7.6 sec.	4.12 - 10	0%	4 sec.
K	1	0.7	4.13 - 1	0%	12.4 sec.	4.13 - 2	0%	4.4 sec.
K	1	0.5	4.13 - 3	0%	19.4 sec.	4.13 - 4	0%	5.6 sec.
ω_n^2	1	1.1	4.13 - 5	4%	9.6 sec.	4.13 - 6	0%	5.6 sec.
ω_n^2	1	1.3	4.13 - 7	14%	13 sec.	4.13 - 8	1%	5.6 sec.
ω_n^2	1	1.5	4.13 - 9	25%	16 sec.	4.13 - 10	1%	5.6 sec.
ω_n^2	1	0.9	4.14 - 1	4%	6.8 sec.	4.14 - 2	0.5%	4 sec.
ω_n^2	1	0.7	4.14 - 3	14%	19.6 sec.	4.14 - 4	4%	8.8 sec.
ω_n^2	1	0.5	4.14 - 5	24%	32 sec.	4.14 - 6	12%	14 sec.

Parameter Changed	Original Value	New Value	Conventional feedback: Fig.	Conventional feedback: Max. overshoot/ undershoot	Conventional feedback: Settling time	Conditional feedback: Fig.	Conditional feedback: Max. overshoot/ undershoot	Conditional feedback: Settling time
ζ_p	0.5	0.55	4.14 - 7	3%	9.8 sec	4.14 - 8	0.5%	4 sec.
ζ_p	0.5	0.65	4.14 - 9	7.5%	12 sec.	4.14 - 10	2%	7.2 sec.
ζ_p	0.5	0.75	4.15 - 1	10.5%	16 sec.	4.15 - 2	3.5%	7.2 sec.
ζ_p	0.5	0.45	4.15 - 3	4%	11.4 sec.	4.15 - 4	1%	8.2 sec.
ζ_p	0.5	0.35	4.15 - 5	13%	20 sec.	4.15 - 6	1%	7.2 sec.
ζ_p	0.5	0.25	4.15 - 7	27%	36 sec.	4.15 - 9	1.5%	9.6 sec.
A step disturbance of 5 V is applied at the output			4.15 - 9	-	4 sec.	4.15 - 10	-	0.6 sec.

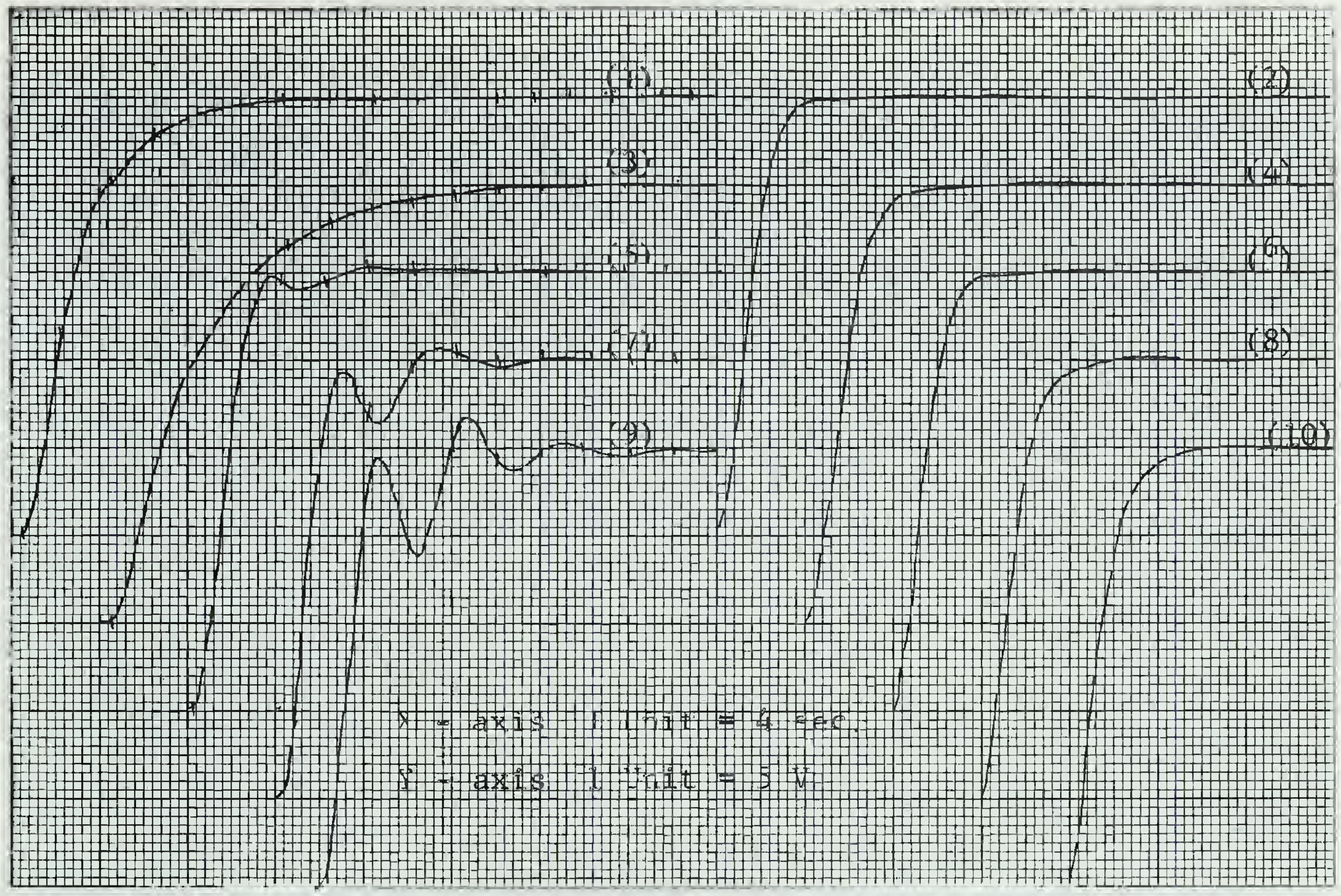


Fig. 4.13

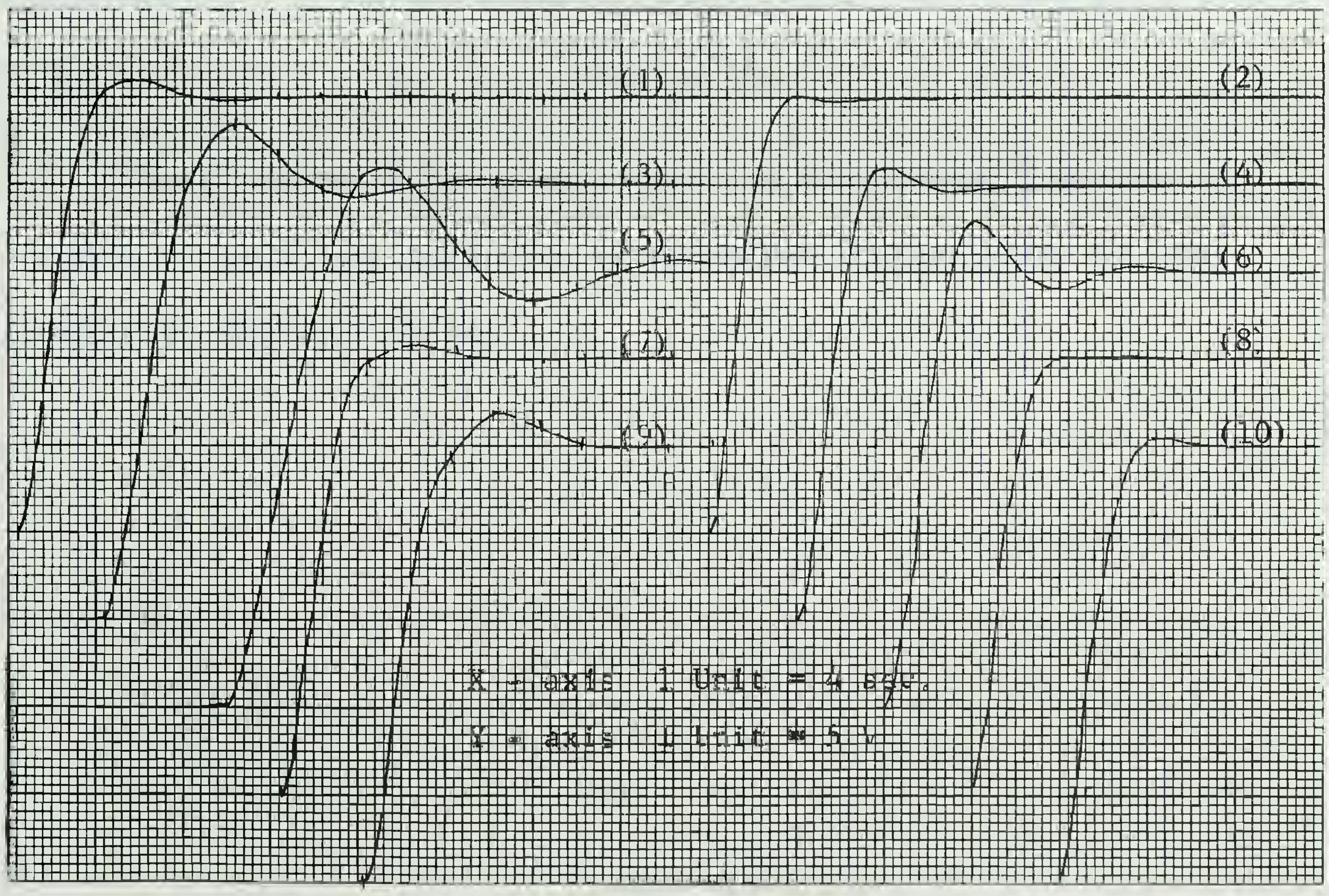


Fig. 4.14

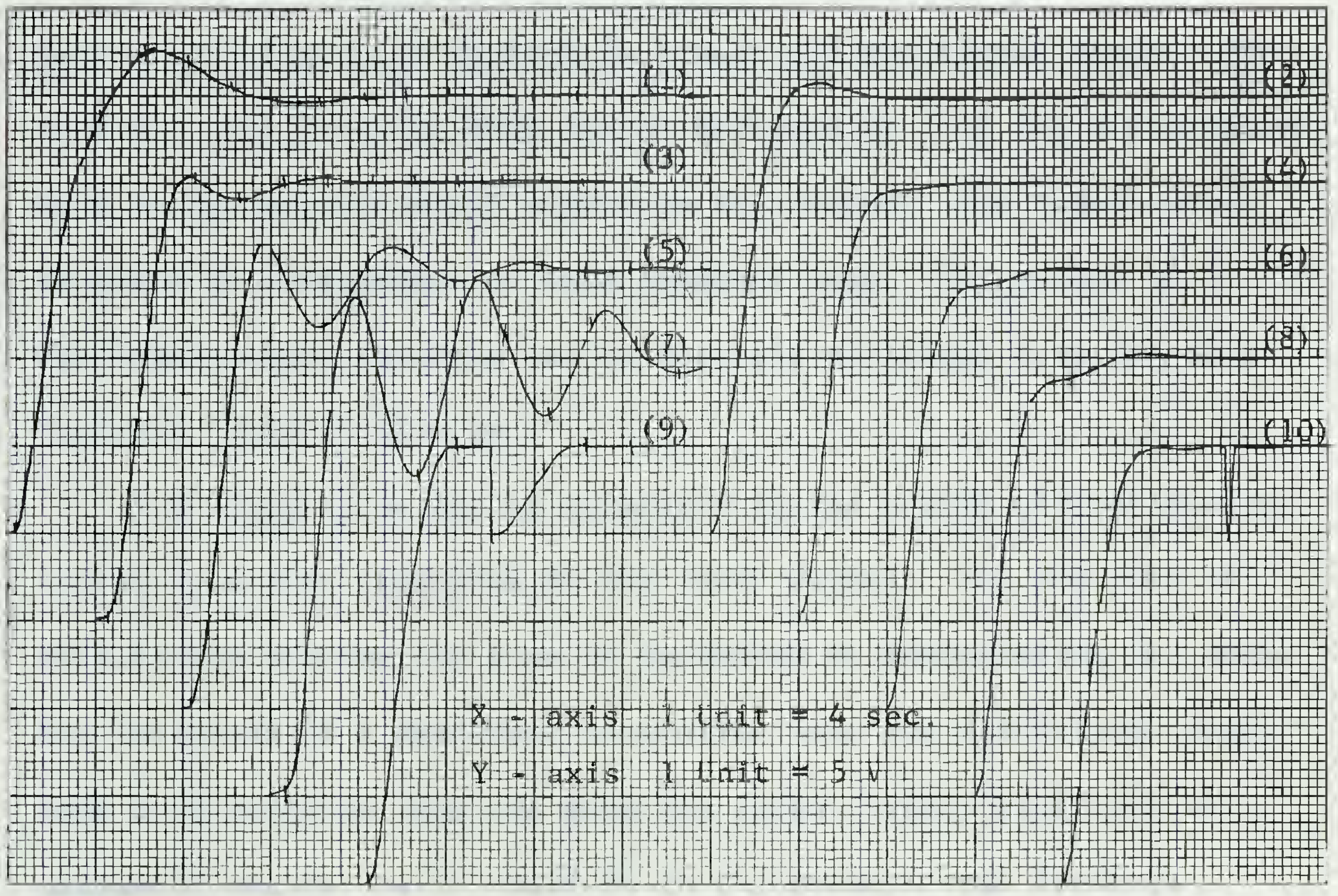


Fig. 4.15

These results clearly show that the conditional feedback system is much less sensitive to parameter changes than the conventional feedback system.

CONCLUSION

This thesis indicates that the idea of conditional feedback can be advantageously applied to linear and non-linear sampled data systems. The feedback loop can be designed so that the disturbances have an appreciably small effect on the output. The input-output response is independent of this feedback loop. The nature of the input-output response can be entirely different from the disturbance- output response.

Non-linearities can be tolerated in the system without causing instability with reference to the input signal. The response of oscillatory systems can also be optimised by manipulating the model. The inclusion of a dead-beat controller into the conditional feedback configuration does not make the output highly sensitive to parameter changes as it would do in the conventional feedback configuration.

REFERENCES

1. G.Lang, J. M. Ham, "Conditional Feedback Systems - A New Approach to Feedback Control", A.I.E.E. 55 - 202, No. 19, July 1955, pp. 152 - 161.

2. B. C. Kuo, "Analysis and Synthesis of Sampled Data Systems", (Englewood Cliff's N. J. Prentice Hall, Inc., 1964).

3. D. R. Katt, "Conditional Feedback Systems applied to Stabilizing a Missile in Pitch Attitude", WESCON Convention Record, 1957, pp. 247 - 253.

APPENDIX A

REPETITIVE OPERATION METHOD FOR DESIGNING THE DEAD-BEAT CONTROLLERS (REP-OP METHOD)

This is an analog computer method for designing the dead-beat controllers for linear and non-linear systems. This method was developed by Professor Y. J. Kingma.

Fig. A.1 shows a closed loop control system.

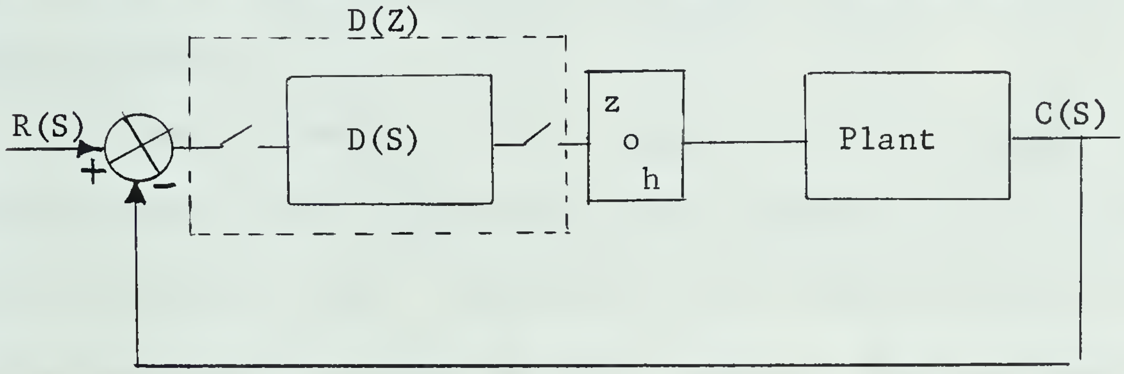


Fig. A.1

The controller D(S) is designed to give a dead-beat response. When the input is applied to the system, the digital controller produces a series of step inputs such that the response of the system is optimum. The duration of each step input is T. The number of these steps depends upon the order of the system, the type of the system and the type of the input to the system. The digital controller can be expressed in the z-domain as follows.

$$D(Z) = \frac{ho + h_1Z^{-1} + h_2Z^{-2} + h3Z^{-3} + \text{-----}}{eo + e_1Z^{-1} + e_2Z^{-2} + e3Z^{-3} + \text{-----}}$$

If this is analyzed by the state Transition Technique, it is found that ho, h_1, h_2, h_3 ----- etc. are the magnitudes of the step inputs

to the plant at t = 0, t = T, t = 2T, t = 3T ------ etc. sampling instants respectively and e0, e_1, e_2, e_3 ------ etc. are the magnitudes of the signals that are fed into the controller at the sampling instants t = 0, t = T, t = 2T, t = 3T ----- etc. respectively. Thus if the magnitudes of the step inputs into the plant and the magnitudes of the signals into the controller at the sampling instants are known the dead-beat controller can be written down. This information can be obtained by the rep-op method. The procedure is explained below.

The transfer function of the plant is simulated on the analog computer without the feedback loop. The computer is set in the repetitive operation mode. The input to the system is applied from the special equipment available for this purpose. This equipment is shown in fig. A.2. The box shown in fig. A.2 gives a series of steps as shown in fig. A.3. The magnitude and the sign of each of these steps is controlled by a separate potentiometer and a switch. The output of the plant transfer function at various locations is displayed on a multi-channel oscilloscope. The outputs appear stationary on the screen because the computer is in the high speed repetitive operation mode. The potentiometers and the switches are then adjusted till all the outputs meet the requirements of dead-beat response. The settings of the potentiometers and the sign of the switches give the value of the h's, and the values of the e's can be read off on the oscilloscope. The dead-beat controller can then be written down.

The box in fig. A.2 can give five steps, the maximum value of each step is 11.0 volts. The sampling period can be adjusted by the

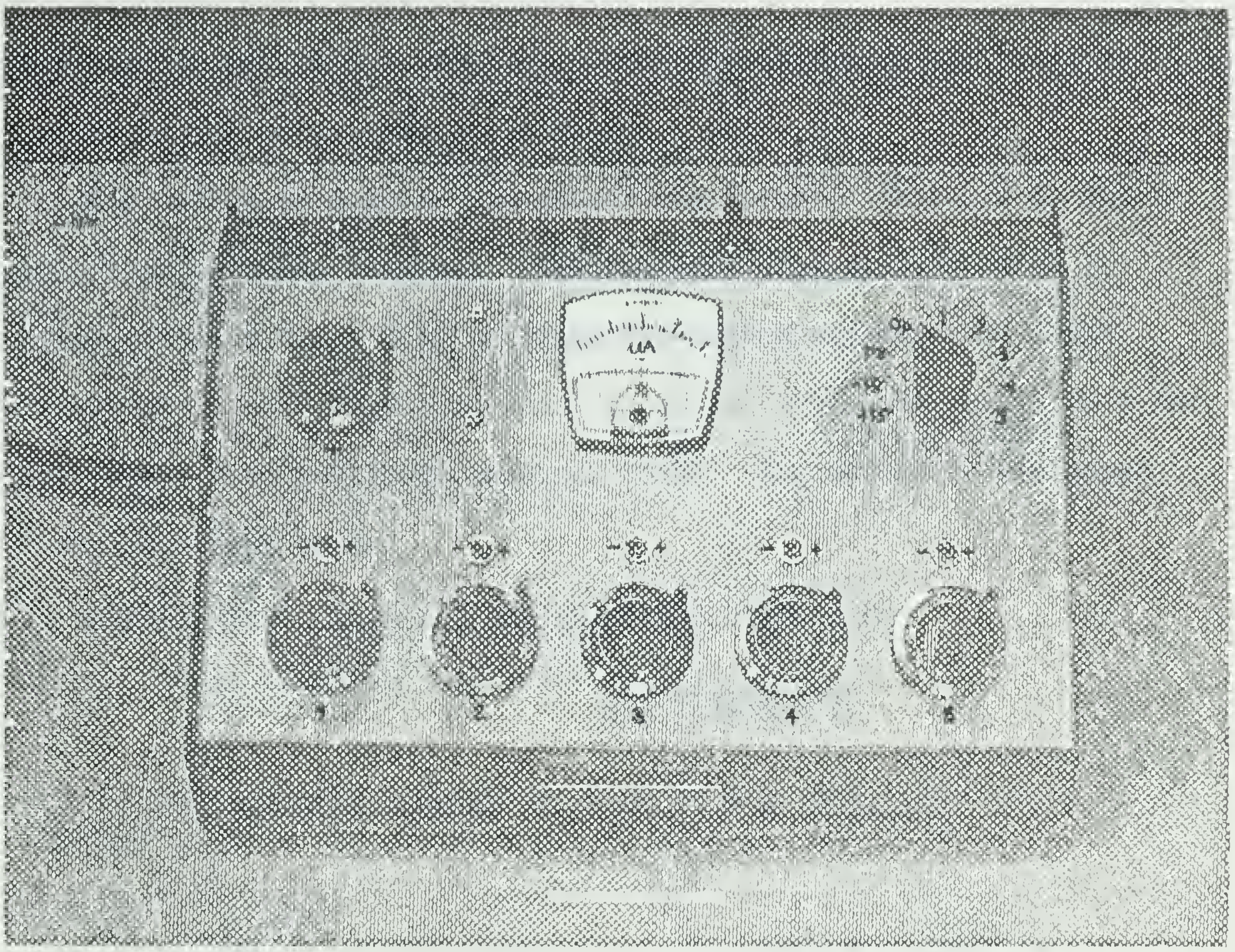


Fig. A.2

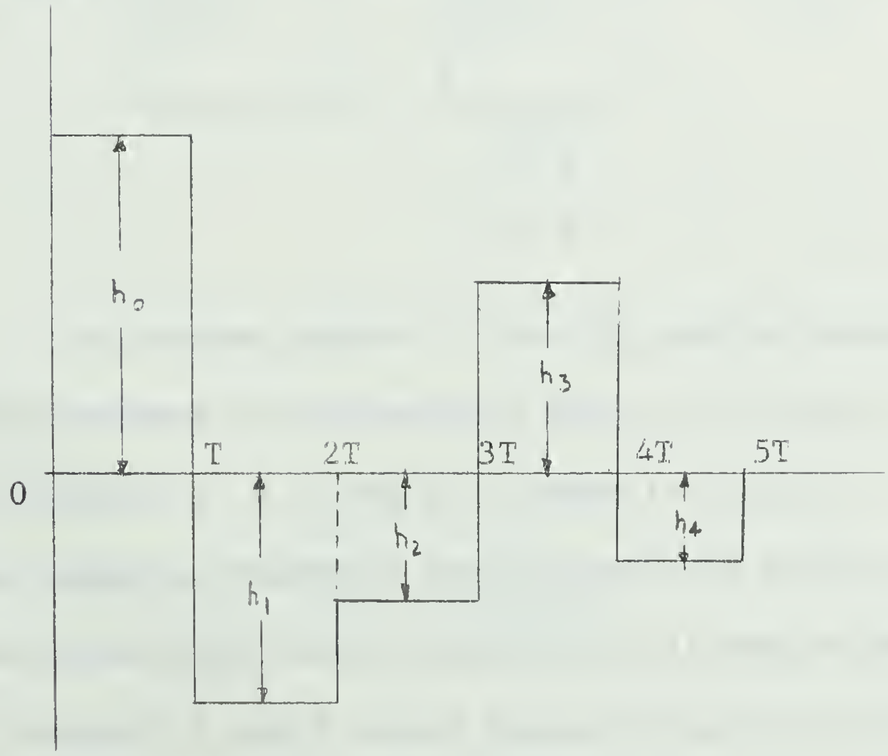


Fig. A.3

potentiometer marked T in fig. A.2. The magnitudes of the steps is adjusted by the potentiometers numbered 1 to 5.

This method is simple, convenient and direct. There is no inter-sample ripple in the response. The presence of a non-linearity or a transportation delay does not make the method any more difficult. This method is also very convenient to design dead-beat controllers for plants which have auxiliary feedback loops. The method is explained for the following cases:

A.1 <u>A third order, type one system with a step input</u>

$$\text{Plant transfer function} = \frac{1}{S(S+1)(S+2)} .$$

The computer diagram is shown in fig. A.4.

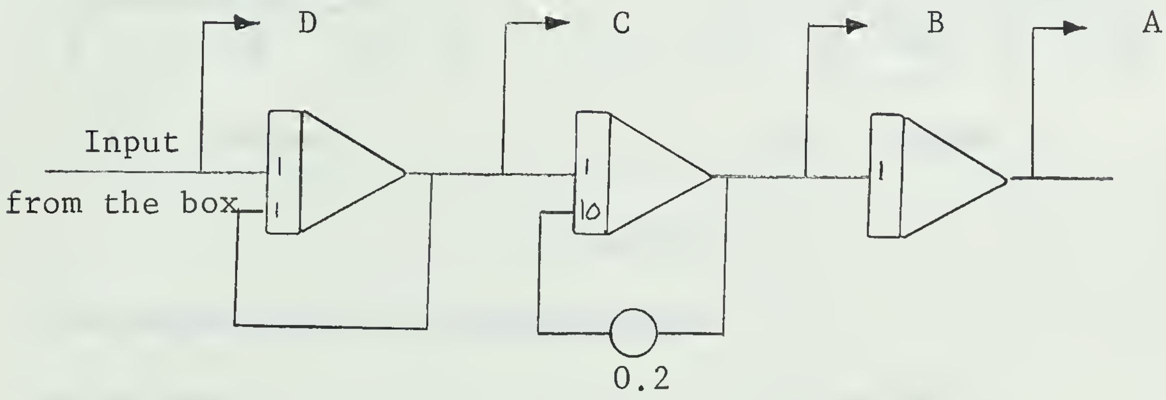


Fig. A.4

The minimum number of sampling periods required for dead-beat response in this case is three. The form of the signals at channels A, B, C and D is shown in fig. A.5. At t = 3T, the signal at channel A should reach the value of the step input and after that it should stay at that value. The signals at channels B and C should reduce to zero at t = 3T and stay at zero after that.

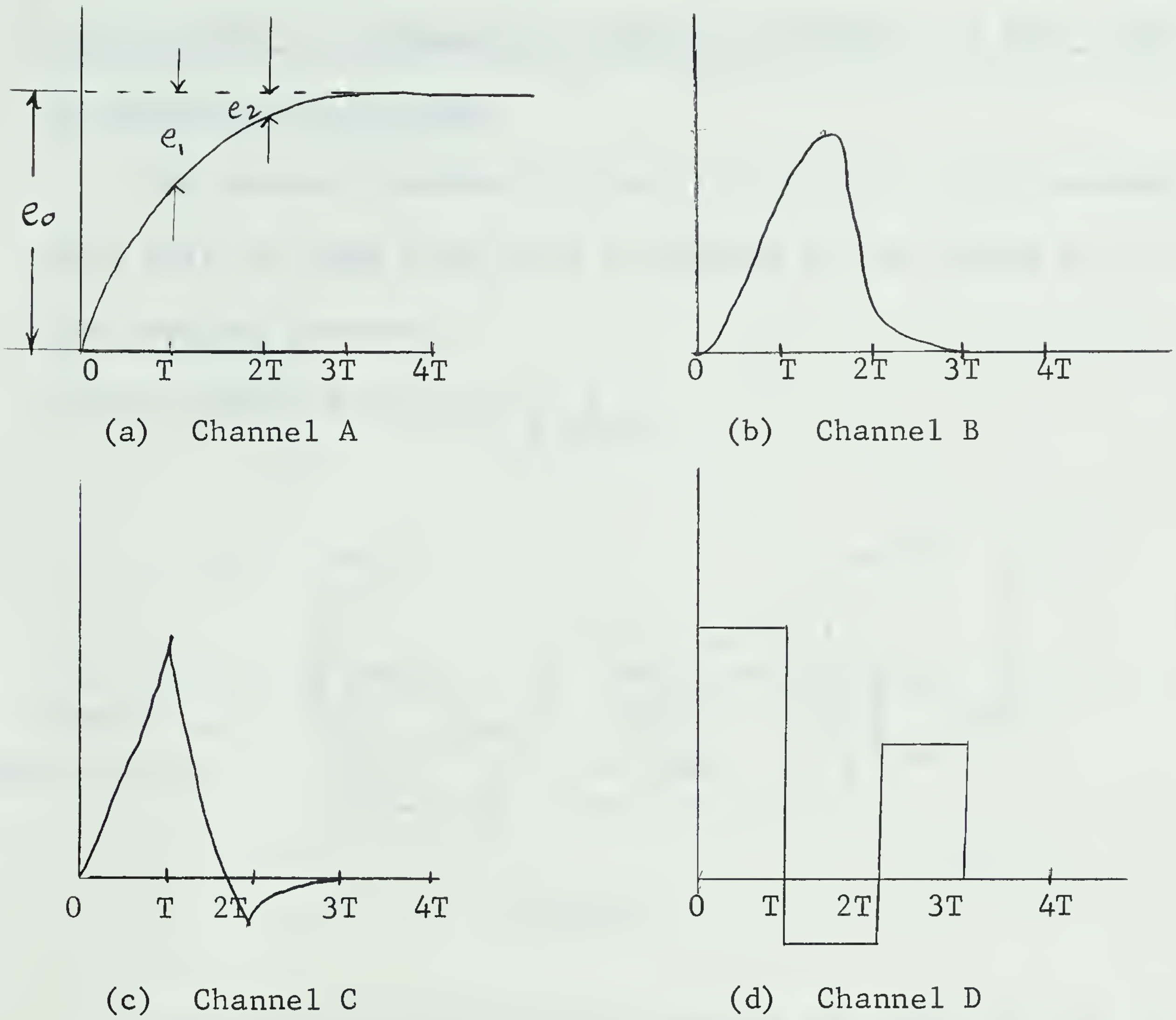


Fig. A.5

The values found in this case were

$e_o = 14.41$ $\qquad$ $h_o = +50$

$e_1 = 9.88$ $\qquad$ $h_1 = -25.1$

$e_2 = 1.15$ $\qquad$ $h_2 = +2.5$

The dead-beat controller will be

$$D(Z) = 3.47 \frac{1 - 0.502Z^{-1} + 0.05Z^{-2}}{1 + .685Z^{-1} + 0.0798Z^{-2}}$$

A.2 <u>A second order, type one system containing a saturation type non-linearity in between the transfer functions. A ramp input is applied to the system</u>

The computer diagram is shown in fig. A.6. It is assumed here that the ramp input will be applied to the system at one of the sampling instants.

Plant transfer function = $\dfrac{1}{S(S + 1)}$

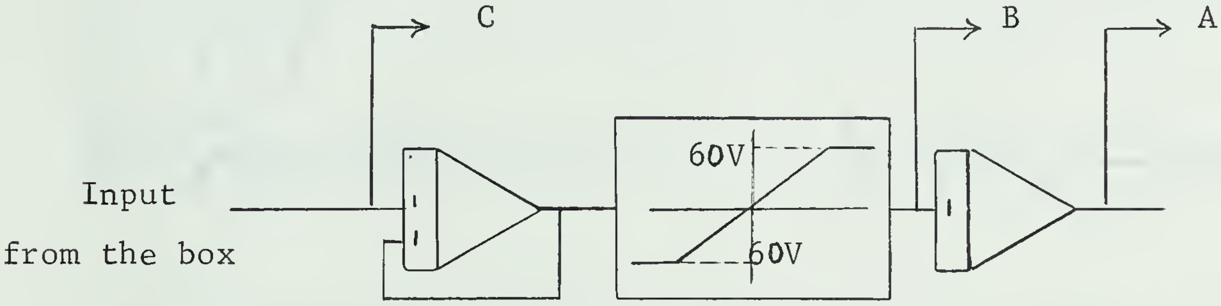
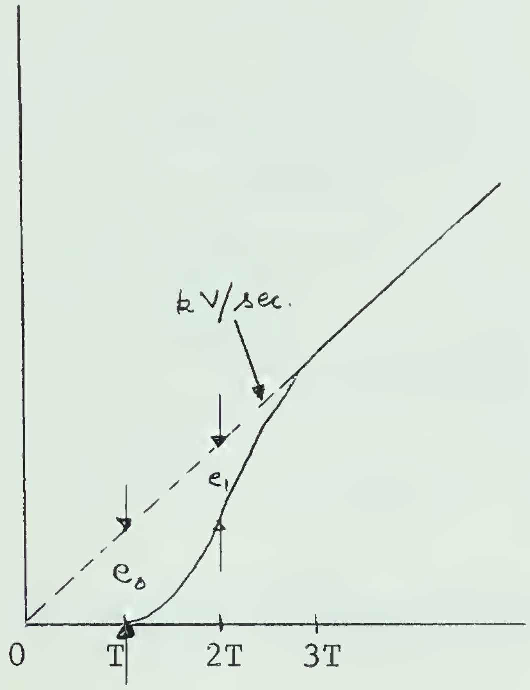


Fig. A.6

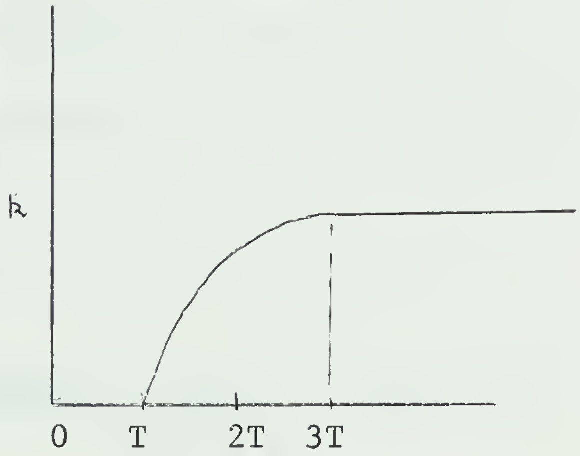
The signals at the various channels are shown in fig. A.7.

No signal will appear at the input to the plant at the first sampling instant because the input is a ramp and there is a sampler and a zero order hold before the plant.

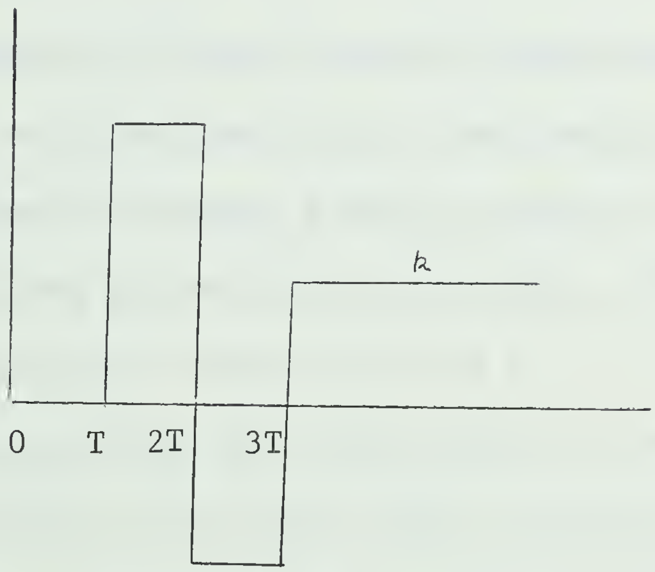
In this case it is required that the controller should also act as an integrator i.e. it should hold the value at the output when the input to it reduces to zero. This is necessary because the integrator in the plant will give a ramp output only when it has a constant step input to it. When the ramp output equals the ramp input (dead-beat response), the signal in the error branch will be zero and so will be the input to the controller. This integrating effect can be achieved by associating a digital



Channel A



Channel B



Channel C

Fig. A.7

integrator of the form shown in fig. A.8 with the digital controller.

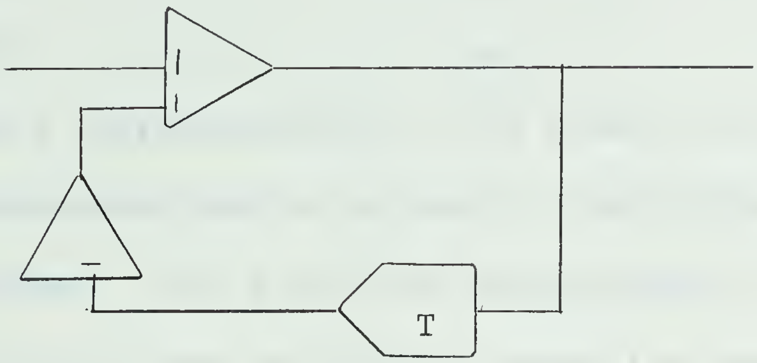


Fig. A.8

The digital integrator can be expressed as $\frac{1}{(1 - Z^{-1})}$. This will be multiplied into the D(Z) found by the above method.

The requirements of dead-beat response in this case are as follows

Channel A: - The signal at channel A should acquire the value of the ramp input at t = 3T and then stay at that value after that.

Channel B: - The signal at channel B should acquire a constant value at t = 3T and stay at that value after that. If the ramp input is k V/sec. then the constant value is k.

Channel C: - In this case the last step applied to the system should be of magnitude k, if the ramp input is k V/sec. This last step, in the above case, will be applied at t = 3T. This is so, because at t = 3T, the input to the controller will reduce to zero and then the digital integrator will hold the value of the last step. This last step will be an additional step. The error will have already been reduced to zero at t = 3T. The

information obtained will be of the following form

$e_o = \ldots$ $h_o = \ldots$

$e_1 = \ldots$ $h_1 = \ldots$

$e_2 = 0$ $h_2 = \ldots$

In a digital controller, the powers of the numerator and the denominator have to be equal. Thus in the above case e_2 is required. This e_2 will be obtained when the digital integrator is multiplied into the digital controller obtained by the above insufficient information. The modified controller will be

$$D(Z)_m = \frac{h_o + h_1 Z^{-1} + h_2 Z^{-2}}{(e_o + e_1 Z^{-1})(1 - Z^{-1})} \qquad \text{A.1}$$

The digital integrator inserted between the dead-beat controller and the plant will modify the steps being produced by the controller. This modification can be explained as follows. From equation A.1, the steps produced by the digital controller are h_o, h_1 and h_2. The digital integrater is shown in fig. A.8. When the step inputs to this integrator are h_o, h_1 and h_2 at the instants $t = 0$, $t = T$ and $t = 2T$ respectively, the outputs of the integrator will be

h_o at $t = 0$ A.2

$h_o + h_1$ at $t = T$ A.3

$h_o + h_1 + h_2$ at $t = 2T$ A.4

But to get the dead-beat response from the plant, the step inputs into the plant should be h_o, h_1 and h_2 at the instants $t = 0$, $t = T$ and $t = 2T$ respectively. This can be achieved by modifying

the step outputs of the controller. Let the modified step outputs of the controller be

at $t = 0 \rightarrow h_o'$

at $t = T \rightarrow h_1'$

at $t = 2T \rightarrow h_2'$

Solving equations A.2, A.3 and A.4 for h_o', h_1' and h_2' yields,

$h_o' = h_o$

$h_1' = h_1 - h_o$

$h_2' = h_2 - h_1$

Therefore the modified dead-beat digital controller will be

$$D(Z)_m = \frac{ho' + h_1'Z^{-1} + h_2'Z^{-2}}{(e_o + e_1Z^{-1})(1 - Z^{-1})}$$

$$= \frac{h_o' + h_1'Z^{-1} + h_2'Z^{-2}}{e_o + (e_1 - e_o)Z^{-1} + e_1Z^{-2}}$$

It may be noted here that the sum of the coefficients of the powers of Z in the denominator will always be equal to zero.

For the plant considered above the values found were:

ramp input = 20 V/sec.

saturation limit of the non-linearity = 60 V

$e_o = +20$	$h_o = +78.43$
$e_1 = +10$	$h_1 = -1.03$
$e_2 = 0$	$h_2 = +20$

Hence

$h_o' = +78.43$ $\quad$ $h_2' = + 21.03$

$h_1' = -79.46$

$$D(Z)_m = \frac{78.43 - 79.46Z^{-1} + 21.03Z^{-2}}{(20 + 10Z^{-1})(1 - Z^{-1})}$$

$$= \frac{78.43 - 79.46Z^{-1} + 21.03Z^{-2}}{20 - 10Z^{-1} - 10Z^{-2}}$$

A.3 A second order type zero oscillatory system, with a step input

The plant transfer function $= \dfrac{1}{S^2 + S + 1}$

The requirements in this case are exactly the same as for the type one system with a ramp input. The controller should act as an integrator in this case also. The computer diagram is shown in fig. A.9.

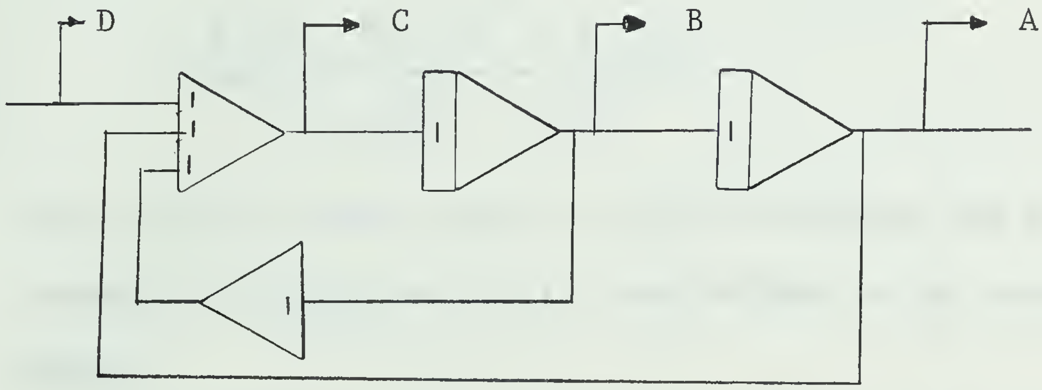


Fig. A.9

The requirements for dead-beat response are

1. The signal at channel A should reach the value of the step input at t = 2T and remain at that value after that.
2. The signals at B and C should reduce to zero at t = 2T and then remain at zero after that.
3. The step output of the controller at t = 2T should be equal

to the step input to the system.

The values found were

$$e_o = 3.0 \qquad h_o = +86.16$$

$$e_1 = 1.47 \qquad h_1 = -66.54$$

$$e_2 = 0 \qquad h_2 = 3.0$$

Hence

$$h_o' = +86.18$$

$$h_1' = -152.72$$

$$h_2' = +69.54$$

$$D(Z)_m = \frac{86.18 - 152.72Z^{-1} + 69.54Z^{-2}}{(3 + 1.47Z^{-1})(1 - Z^{-1})}$$

$$= \frac{86.18 - 152.72Z^{-1} + 69.54Z^{-2}}{3 - 1.53Z^{-1} - 1.47Z^{-2}}$$

Thus it may be seen that it is very convenient and straight forward to design the digital controllers by the rep-op method.

A digital computer program was worked out to check the correctness of the dead-beat controllers designed by the rep-op method. This program is shown in appendix C.

APPENDIX B

This appendix illustrates the simulation of the hysteresis type non-linearity on the analog computer. This scheme minimizes the drift. The arrangement is shown in fig. B.1.

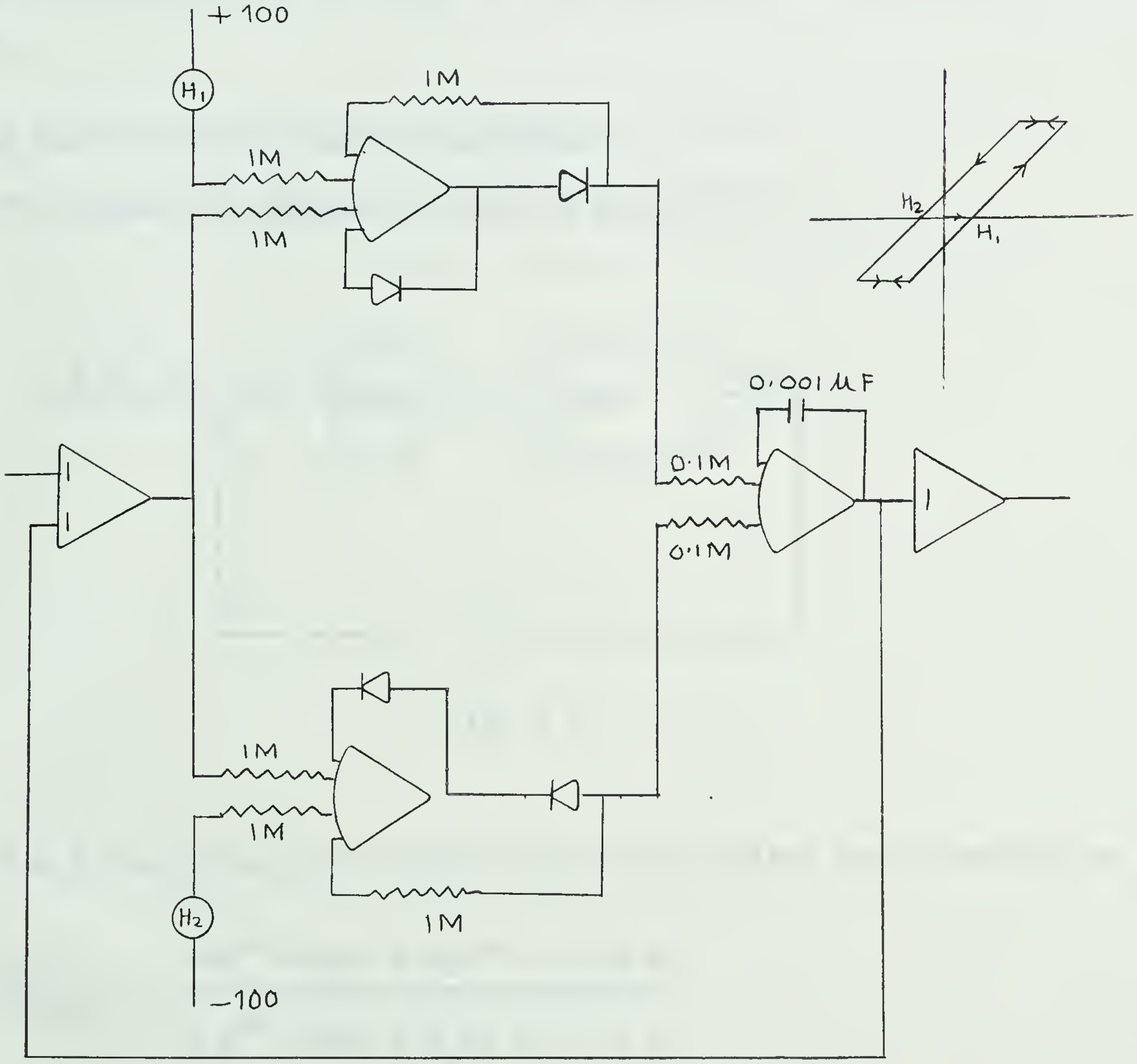


Fig. B.1

APPENDIX C

Digital computer program to check the correctness of dead-beat controllers

This digital computer program gives the values of the output at the sampling instants if the coefficients of the powers of Z in the Z-transforms of the plant and the digital controller are known.

A step input to the system is considered.

The closed loop system is shown in fig. C.1.

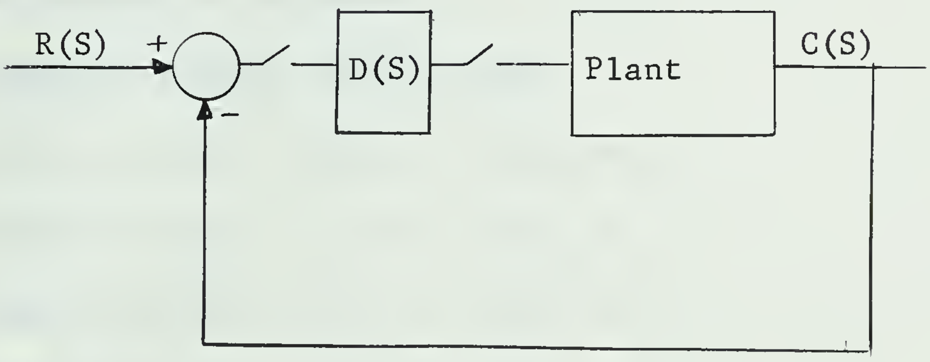


Fig. C.1

The Z-transforms of the controller and the plant can be written as

$$D(Z) = P_1 \frac{D_4Z^4 + D_3Z^3 + D_2Z^2 + D_1Z + D_o}{E_4Z^4 + E_3Z^3 + E_2Z^2 + E_1Z + E_o}$$

$$G_p(Z) = P_2 \frac{G_4Z^4 + G_3Z^3 + G_2Z^2 + G_1Z + G_o}{H_4Z^4 + H_3Z^3 + H_2Z^2 + H_1Z + H_o}$$

$$C(Z) = \frac{D(Z)\ G_p(Z)}{1 + D(Z)\ G_p(Z)} \quad \times \quad R(Z)$$

$$R(Z) = \frac{Z}{Z - 1}$$

$$\therefore C(Z) = \frac{A(I)Z^{10} + A(I - 1)Z^{9} + A(I - 2)Z^{8} + \text{-----}}{B(I)Z^{10} + B(I - 1)Z^{9} + B(I - 2)Z^{8} + \text{-----}}$$

1. The values of D'S, E'S, G'S, H'S, P'S are to be read into the program after the end card. The value of I is also to be read. If,

   m = Max. power of Z in the denominator of D(Z)

   n = Max. power of Z in the denominator of $G_p(Z)$

   then I = m + n + 2 (Maximum value of I for this program is 10)

   The program is as under

```
      DIMENSION Z(100), B(20), A(20)
      READ (5,1)  D0,  D1,  D2,  D3,  D4
      READ (5,1)  E0,  E1,  E2,  E3,  E4
      READ (5,1)  G0,  G1,  G2,  G3,  G4
      READ (5,1)  H0,  H1,  H2,  H3,  H4
      READ (5,2)  I
      READ (5,3)  P1,P2
    1 FORMAT  (1X, 5(F10.7,  1X))
    2 FORMAT  (1X, I2)
    3 FORMAT  (1X, 2(F10.6,  1X))
      A(1)  = + 0.0000000
      A(2)  = P1*P2*D0*G0
      A(3)  = P1*P2*(D1*G0 + D0*G1)
      A(4)  = P1*P2*(D2*G0 + D1*G1 + D0*G2)
      A(5)  = P1*P2*(D3*G0 + D2*G1 + D1*G2 + D0*G3)
```

```
A(6) = P1*P2*(D4*G0 + D3*G1 + D2*G2 + D1*G3 + D0*G4)

A(7) = P1*P2*(D4*G1 + D3*G2 + D2*G3 + D1*G4)

A(8) = P1*P2*(D4*G2 + D3*G3 + D2*G4)

A(9) = P1*P2*(D4*G3 + D3*G4)

A(10) = P1*P2*D4*D4

C0 = E0*H0 + A(2)

C1 = (E1*H0 + E0*H1 + A(3))

C2 = E2*H0 + E1*H1 + E0*H2 + A(4))

C3 = (E3*H0 + E2*H1 + E1*H2 + E0*H3 + A(5))

C4 = (E4*H0 + E3*H1 + E2*H2 + E1*H3 + E0*H4 + A(6))

C5 = (E4*H1 + E3*H2 + E2*H3 + E1*H4 + A(7))

C6 = (E4*H2 + E3*H3 + E2*H4 + A(8))

C7 = (E4*H3 + E3*H4 + A(9))

C8 = (E4*H4 + A(10))

B(1) = -C0

B(2) = C0 - C1

B(3) = C1 - C2

B(4) = C2 - C3

B(5) = C3 - C4

B(6) = C4 - C5

B(7) = C5 - C6

B(8) = C6 - C7

B(9) = C7 - C8

B(10) = C8

Z0 = A(I)/B(I)

Z(1) = (A(I - 1) - Z0*B(I - 1))/B(I)
```

```
40  J = I - 1
    DO 60 K = 2,J
    Z(KK) = 0.0
    JJ = K - 1
    DO 61 N = 1,JJ
    MM = I - K + N
61  Z(KK) = Z(KK) + Z(N)*B(MM)
    LLL = I - K
60  Z(K) = (A(LLL) - Z0*B(LLL) - Z(KK))/B(I)
50  L = I - 1
    DO 20 K = 1,35
    LL = L + K
    Z(LL) = 0.0
    II = I - 1
    DO 21 N = 1,II
    M = N - 1 + K
21  Z(LL) = Z(LL) + Z(M)*D(N)
20  Z(LL) = -(Z)LL))/B(I)
    WRITE (6,4) Z0
    DO 25 I = 1,43
25  WRITE (6,4) Z(I)
 4  FORMAT (5X,F10.5)
    WRITE (6,6)
 6  FORMAT (1H1,5X,14HVALUES OF A(I))
    DO 26 I = 1,N
26  WRITE (6,4) A(I)
```

```
      WRITE (6,7)
 7    FORMAT (1H1,5X,14HVALUES OF B(I))
      DO 27 I = 1,10
27    WRITE (6,4) B(I)
 5    FORMAT (10X,F10.5)
      END
```

Example

A second order oscillatory system with the transfer function $\dfrac{1}{S^2 + S + 1}$ was chosen

$$G_p(Z) = 0.0187 \, \frac{Z + 0.9251}{Z^2 - 1.783Z + 0.819}$$

The dead-beat controller found by the rep-op method was

$$D(Z) = 14.25 \, \frac{1 - 0.7988Z^{-1} - 0.9270Z^{-2} + 0.798Z^{-3}}{1 - 0.25Z^{-1} - 0.5Z^{-2} - 0.25Z^{-3}}$$

T = 0.2 sec.

I = 7

Input to the system = a unit step U(t)

The results found were as follows:

P.T.O.

VALUES OF THE OUTPUT AT THE SAMPLING INSTANTS

t = 0	0.00000	t = 20T	0.99837
t = T	0.26647	t = 21T	0.99859
t = 2T	0.77087	t = 22T	0.99882
t = 3T	1.00519	t = 23T	0.99905
t = 4T	1.00238	t = 24T	0.99927
t = 5T	1.00520	t = 25T	0.99948
t = 6T	1.00603	t = 26T	0.99966
t = 7T	1.00546	t = 27T	0.99983
t = 8T	1.00326	t = 28T	1.00009
t = 9T	1.00208	t = 29T	1.00018
t = 10T	1.00103	t = 30T	1.00025
t = 11T	0.99940	t = 31T	1.00029
t = 12T	0.99883	t = 32T	1.00032
t = 13T	0.99840	t = 33T	1.00033
t = 14T	0.99811	t = 34T	1.00033
t = 15T	0.99795	t = 35T	1.00031
t = 16T	0.99789	t = 36T	1.00029
t = 17T	0.99792	t = 37T	1.00026
t = 18T	0.99802	t = 38T	1.00023
t = 19T	0.99818	t = 39T	1.00019

VALUES OF A(I)

A(1)	0.00000	A(6)	0.26647
A(2)	0.19665	A(7)	0.00000
A(3)	-0.01595	A(8)	0.00000
A(4)	-0.44394	A(9)	0.00000
A(5)	0.03366	A(10)	0.00000

VALUES OF B(I)

B(1)	0.00810	B(6)	-2.76652
B(2)	-0.02840	B(7)	1.00000
B(3)	0.02749	B(8)	0.00000
B(4)	-0.80560	B(9)	-0.00000
B(5)	2.56493	B(10)	0.00000